PREFACE

This book is about the solution to and properties of the Coupled Exponent equation (y=x^x). The solution to this equation is called the "Coupled Root function". This work details our research efforts since 1975. Included are computers/calculators used, evolution of ideas, history of our efforts and still outstanding problems. We have organized the work into different topics such as "Applications", "Solving logarithmic Equations", "Integration", etc. to make it easier for the reader to find a topic. This is a work where the appendices and tables are (in some ways) more important then the text itself. The text is to explain the theory; the tables have the actual items of interest.

Our goal in writing this book is to show the (in our opinion) interesting things we found and to encourage research into this topic as we feel this is one area that has been mostly overlooked. We feel that the Coupled Root function has many hidden properties that have the potential to be useful. Two such applications have been found so far: Ballistics (internal & external) and automobile acceleration. There is no doubt other areas where the Coupled Root could be used.

What got us interested in the Coupled Exponent as to want to research it more? In 1974, we were in junior high school (7-9 grade) and calculators have just dropped in price just enough to make them within reach of us to buy. After spending our entire lives doing arithmetic "by hand", we were delighted to have a machine that would perform arithmetic correctly out to 8 decimal places. We had learned to use slide rules, but they were accurate to 3 digits and they did not present their answers in large bright red or green LED displays. We learned that computers (in those days they were large metal boxes with tape drives and blinking lights and were *very* expensive - off limits to all non-experts!) used powers of 2 (binary numbers). From this we would calculate the powers of 2 on the calculator by entering 2 and then pressing the "multiply" key followed by "=" key. Repeated pressing of "=" would give the sequence:

2, 4, 8, 16, 32, 64, 128, 256, 512, 1024, 2048, 4096, ...

It was fun to see how far this would go before the calculator "overflowed" i.e. locked-up until the CLEAR key was pressed to reset the machine. We knew that in our sequence, each element was twice the size of the one before it. We learned also of the factorial sequence,

1, 2, 6, 24, 120, 720, 5040, 40320, 362880, ...

and found this grew faster than the binary sequence.

The American mathematician Philip J. Davis wrote a book for non-math experts called "The Lore of Large Numbers". In this work, Professor Davis discusses sequences like the sequence of squares and cubes and the factorial. In the appendix of his book was a chart comparing different sequences. Like most readers, we wanted to know what the fasting growing sequence was and cared little about the others. The fastest sequence was labelled "The Coupled Exponentials" and the sequence given was,

1, 4, 27, 256, 3125, 46656, 823543, 16777216, 387420489, 1.0E+10

It was very impressive to see how quickly the numbers grew. We saw something new that had a distinct "pull" to it.

We know how to invert squares (computing square roots) and invert binary sequences (by computing log2(x)) but we didn't know how to invert the Coupled Exponentials. We could solve,

x^2 = 10   x = 3.16227766

$x^3 = 10 \qquad x = 2.15443469$

$2^x = 10 \qquad x = 3.321928095$

but we could not solve

$x^x = 10 \qquad x = ?$ (we know it was between 2 and 3)

We asked our math teachers about this and they were unable to solve this and told us there was no quick & easy solution. From this, we entered the world of mathematical research.

Over the years, our efforts were directed along numeric lines. We wanted a way to compute Coupled Roots as quickly and as painless a way as possible. Calculators have advanced to the point where it was possible for one of us to obtain a programmable calculator with LCD display (no more dead batteries every two hours) and constant memory. When the machine was turned off, it still "remembered" the program and data that had been entered into it. Most scientific calculators have a dynamic range of 10^{100}. That is, the biggest number that can be entered is $9.999999*10^{99}$. From this, the biggest number we could compute the coupled root of was 10^{100}. This value is = 56.96124843 We wanted to know things like the coupled root of 10^{1000}. By summer 1980, we found that coupled roots had "quasi-logarithmic" properties. By this we mean that coupled roots "acted" like logarithms when the argument to the coupled root was large. Earlier that summer, one of us computed the integral of coupled root of e^x in closed form. Efforts to compute either a coupled exponent or coupled root integral in terms of elementary functions resulted in failure.

In 1981, the term "Wexzal" was defined to mean "Coupled Root of 10^x". With this new notation, the manipulation of coupled roots of large numbers was made easier. By end of February 1981, the asymptotic property of the Wexzal was discovered and proved. This explained the earlier "quasi logarithmic" behavior that was observed.

Thru the 1980's the Wexzal was researched in great detail. New integrals that could be written in closed form (involving the Wexzal) were found. An asymptotic expression for inverting factorials was also found. New properties (mostly involving asymptotics) were found. Logarithmic equations were solved in terms of the Wexzal. Super fast means of computing Wexzals on programmable calculators was developed. Methods were developed for approximating Wexzals on 4-function calculators as these machines (sometimes known as "4-bangers") were cheap and quickly obtainable. Numerical methods were developed for computing least squares that involved the Wexzal.

Because of the rapid growth of coupled exponentials, it became clear that accuracy/precision was of upmost importance. This lead us to define very high computing standards for calculators and computers. The reason for the focus on calculators during the time when early home computers were making their appearence is that the early home computers used the BASIC language which supported only 6-7 digits of precision and was found to not be too useful for Wexzal work despite having greater speed and memory than any calculator that we had.

By 1990 we had amassed a collection of over 200 results (integrals, asymptotics, closed form solution to equations, etc) involving coupled roots and Wexzals. We found that we had not looked (too hard) into applying this work.

That changed quickly when a friend of one of the authors asked him to "come-up with a formula that relates barrel length of a gun to the muzzle velocity for a given bullet & powder charge". This question along with a related one involving velocity decay for high speed projectiles helped to redirect our research efforts from questions of theoretic interest to those of more practical nature. It was expected that both of these questions could be quickly solved using classical methods involving exponential/logarithmic equations but these were found wanting for the degree of precision desired.

It was discovered in 1993 that for high speed projectiles having speeds of over 1370 ft/sec, the velocity decay (v=f(x) where v is velocity and x is distance) can be described with high accuracy with an equation involving the Wexzal function. This formula agrees with values found in standard ballistic tables (Ingall's [USA] and Krupp's tables [Germany]). The discovery that nature can be modeled with non-classical equations is, to us, amazing and leads one to wonder if more events in nature can be better described with non-classical equations. Today, with the advent of super fast small computers, we expect all areas of physics and computational mathematics to undergo a re-evaluation as the types of methods and equations used for research. Nature is not as simple as we think.

Are we the first to research Coupled Roots and related functions? No. Giants of mathematics such as Euler, Eisenstein, Lambert, Hardy and others have touched on Coupled Roots and Coupled Exponents. Today, our "competition" would be Professors Corless, Gonnet, Hare and Jeffrey of University of Waterloo (Canada) who have written the paper "On Lambert's W Function". Johann Heinrich Lambert [1728-1777] was a German mathematician who research many areas of mathematics. Today, he would be known as an applied mathematician. Part of his research involved solving the equation x=y*exp(y) which the Professors, cited before, chose to name the "W" function. They chose the name W because it looks like the lowercase Greek Omega.

Their 30 page paper is more theoretic than our work here. Their aim was to present the W function in a crisp, concise manner. Because they used an equation involving 'e' as a base instead of 10, one will find that their equations and derivations are "cleaner" than ours. We feel that our "convention" is better suited for applications even at the expense of more complex formulae.

Since the early 1990's there has been much written in both the technical and lay press about the INTERNET, the electronic network that connects computers of all types all over the world. At one time, it was the exclusive domain of scientists, mathematicians and other researchers. Today it is also accessed by the interested lay public. This aids in the free flow of all types of information planet-wide.

We chose to "publish" this work on the INTERNET as we feel that the INTERNET is the way information will be desiminated in the 21st century. This agrees with our goal of encouraging Coupled Root research.

The style of writing found on the INTERNET (based on postings we have seen in SCI.MATH, SCI.PHYSICS, etc.) is informative, entertaining and very informal. Our aim is to have our work be along the same lines. This is not a traditional mathematics textbook but rather an informal reporting of our results. We assume that the reader is knowledgable about Numerical Analysis, Integral & Differential Calculus and Numerical Computing in a scientific computer language such as FORTRAN, BASIC or Ada. For other items (such as guns and gunpowder in our Internal Ballistics chapter) we explain the basics of that topic so the reader can better understand the technical issues involved without having to become an expert. We also chose to present our work in ASCII as to maximize accessibility. This restriction also posed a challenge: There are no diagrams or graphs. The reader is told how to construct these.

All brand names of items noted (computers, cars, guns, etc.) are trademarks of their owners. Names noted do not constitute an endorsement on our part; they are noted to aid others who wish to research further the performance of that device. We are not responsible for others who wish to perform experiments to prove/disprove the validity of our models. We present these results (from a legal standpoint) for "entertainment use only".

For measurements, we have chosen to use the standard U.S. units of measurements. The reason for this is that the experiments/research was conducted in this system. In the U.S. system, confusion sometimes occur between mass and weight. For this work, mass is measured in Slugs and weight (force) is in Pounds. Distance is in Feet and time is in Seconds. Conventional units are given as part of the discussion to aid the reader.

For each chapter, equations are numbered by chapter and actual number.

E.g. (04.12) means equation #12 in Chapter 4. References are noted the same way except brackets are used e.g. [05.02]. References range from common texts to papers found at University Goettingen to private communications to us. Another notation used is for referencing end-of-chapter notes. It is denoted by curly brackets e.g. {11.01}. These notes appear at the end of each chapter and they contain additional information (mostly historical/non-mathematical comments) about the topic to aid understanding. It was set-up this way as to not interupt the flow of the main concept being presented. The interested reader can read the notes later if desired.

We hope the reader finds this work informative and (at the least) entertaining. Please direct all comments and questions to:

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If this work has interested one researcher into researching the questions and topics presented here, then we have met our goal.

The Authors

Chapter 01

The Coupled Exponent

INTRODUCTION

"How quickly does it get large?" This question is asked of number sequences and functions. The sequence (or function) can represent something in nature that increases in size and/or amount or it can be of theoretic value only. The most basic sequence is the linear sequence,

$$1, 2, 3, 4, \ldots \qquad (01.01)$$

where the next term (number) is one more than the one before it. Linear growth is noted by a constant difference between a given term and the term before it.

Another sequence is the sequence of squares (numbers multipled by themselves). This denotes the increase in area as the side is increased. The squares are given by,

$$1, 4, 9, 16, 25, 36, 49, 64, 81, 100, \ldots \qquad (01.02)$$

The sequence of cubes (used to denote an increase in volume) is given by,

$$1, 8, 27, 64, 125, 216, 343, 512, 729, 1000, \ldots \qquad (01.03)$$

The question arises: how does one compute the "inverse" of such a sequence? I.e. How does one find the value of a number when squared gives a given number? E.g.

$$x^2 = 10 \text{ , } x = 3.162277660\ldots \qquad (01.04)$$

This action is called computing the square root {01.01}. The sequences as given in (01.02) and (01.03) are called "power" sequences because the next term is raised to a constant power. The inverse of raising to a power is to compute a root. The root is also noted by the reciprocal of the power. E.g. Cube root of X is noted by X^(1/3) where "^" means to "raise to the power of".

Are the power sequences the fastest growing sequence known? No. If we instead of having the exponent be constant and the base vary, we interchange them, we obtain the exponential sequence. The best known exponential sequence is the binary sequence,

$$2, 4, 8, 16, 32, 64, 128, 256, 512, 1024, \ldots \qquad (01.05)$$

This is formed by j = 2^i for i=1,2,3,...

Each term is twice the size of the one before it. This sequence is used in digital computing. Digital computers use a base 2 number system as this represents the on/off nature of electronic circuits {01.02}. The binary sequence is the source of a popular mathematical story about unexpected rate of growth.

The classic story problem of the Indian peasant who, after inventing the game of chess, was asked by the king what he wanted as a reward for inventing such an enjoyable game. The peasant wanted to be paid one grain of wheat for the first (of 64) square of the chess board, two grains of wheat for the second square, four grains for the third square and so on until all 64 squares of the chess board have been filled. The king soon learned of the effect of doubling in short notice!

The powers of 10 are an exponential sequence also. This is not as "interesting" to write (10, 100, 1000, ...) due to the fact that we use the base 10 number system. However, the inverse of this sequence is of utmost interest.

The act of solving an equation such as

$$10^{x} = 2, \; x = 0.3010299957... \qquad (01.06)$$

is how logarithms come about. Logarithms {01.03} are used to solve equations such as (01.06). They were at one time used to aid in multiplication due to the property,

$$\log(x*y) = \log(x) + \log(y) \qquad (01.07)$$

but calculators have, for the most part, done away with this. Logarithms are still used however.

Southern California is known for sun, surf, Hollywood and earthquakes. Earthquakes are measured on the Richter Scale which is a logarithmic scale. Each "click" up the scale (example: 6.0 to 7.0) represents a force that is 10 times stronger. Sometimes after a major earthquake, scientists will "upgrade" or "downgrade" a quake. What they are doing is updating the rating based on more information gathered from other equipment they have in the field. Most of the time, the adjustment is +/- a couple of 10th of an interval (e.g. 7.1 ==> 6.8). How much of a change is this? The Richter difference is 0.3 so we need to compute,

$$10 \hat{} \, 0.3 = 1.995262315... \qquad (01.08)$$

There are two ways to do this: (1) Using a calculator, enter 0.3 then hit the "10^x" key or (2) Try the following approximation: Assuming we can compute square roots, we can use the following facts:

$$SQRT(x) = x \hat{} \, 0.5, \; SQRT[SQRT(x)] = x \hat{} \, 0.25, \qquad (01.09)$$

where each iteration of the square root hints at a binary sequence. We then try to convert 0.3 into binary (find a computer scientist!). We get,

```
0.3 = 0.0100110011... = 1/4 + 1/32 + 1/64 + ...           (01.10)
        ----------
        248136125
           624251  Decimal value read from top to bottom
              862
```

So we get: 10^(1/4) * 10^(1/32) * 10^(1/64) ... = 1.995 ~ 2.0
The quake was half as strong as originally thought.

One property logarithms have that is very important is that when given a logarithm to a base (such as 10) and one wishes to have a logarithm to a different base (such as 2) all one need do is divide by a constant. The three most "popular" bases used are: 10, 2, and 'e'. These are known as "common", "binary" and "natural" logarithms. Base 'e' is used mostly by pure mathematicians as formulae involving logarithms to this base come out "cleaner" (no conversion factors) then with the other bases. The value of e is,

$$\sum_{k=0}^{\infty} \frac{1}{k!} = \frac{1}{1} + \frac{1}{1} + \frac{1}{2} + \frac{1}{6} + \ldots = 2.718281828\ldots \qquad (01.11)$$

To convert between common and natural logarithms, one uses the conversion factor,

$$m = \log(e) = 0.4342944819\ldots \qquad (01.12)$$

Most logarithm tables are computed to the base 10. This is because of our numbering system. The logarithm of a number has two parts: the "Characteristic" and the "Mantissa". The characteristic is the part to the left of the decimal point and the mantissa is to the right of the decimal point. Logarithmic tables are tables of mantissas only. The characteristic can be determined from inspection of the original number. This is done by first writing the number in scientific notation.

$$2000 \Rightarrow 2.0 * 10 \text{ ^ } 3 = 2.0E+03 \qquad (01.13)$$

The characteristic is the power of 10. In our case, it is 3. We then lookup the logarithm of 2.0 in the table to obtain 0.30103. We then add the characteristic to the mantissa to obtain the final result of 3.30103 For numbers less then 1.0 (but greater than 0.0) the charactistic is a negative number. For log(0.002) we would have,

$$\log(0.002) = 0.30103 - 3 = -2.69897 \qquad (01.14)$$

It is customary to write this as,

$$\log(0.002) = 7.30103 - 10 \quad (7 - 10 = -3) \qquad (01.15)$$

so we can find the mantissa in the table as the table contains only positive values. Once we have the logarithm to the base 10 we can convert to another base by computing,

$$\log_b(x) = \frac{\log(x)}{\log(b)} \qquad (01.16)$$

Most of the time, natural logarithms are noted by ln(x) and binary logarithms by lg(x). The notation "log(x)" is used to mean common logarithms and in theoretic work (and calculus texts) natural logarithms. For this work, log(x) means common logarithms.

Another example of a sequence is the factorial, that is, the product of 1,2,3,...n when given n. The factorial is used greatly in statistics to compute the number of different outcomes. The most basic example is the number of different ways to arrange N objects in a line. The answer is N factorial (written N!). Whereas the exponential sequence has the base fixed, the factorial does not have that restriction so for a given base, the factorial will (if given a big enough number) outgrow the exponential sequence (Fig. 01.01).

n	2^n	n!

1	2	1
2	4	2
3	8	6
4	16	24
5	32	120
6	64	720
7	128	5040
8	256	40320
9	512	362880
10	1024	3628800
100	1.26765E+30	9.3248E+157
1000	1.07151E+301	4.0235E+2567

(Fig. 01.01)

At some point, the factorial "overtook" the binary exponential. The factorial is said to be a faster increasing sequence than the binary exponential. Is there a sequence that increases faster than the factorial and if so, what are some of its basic properties? There are many ways to construct a sequence that grows faster than the factorials. All one need do is to multiply (for example) the factorial and exponential functions to get a new sequence. We are thinking of something like the exponentials except both the base *and* the exponent vary at the same rate. This act leads to something new.

THE COUPLED EXPONENT

The exponential function in the form of

$$y = a ^ x \quad \text{for } a>1 \text{ and all } x \qquad (01.17)$$

is used extensively in applied and pure mathematics. The most common form of (01.17) is for the base to equal e (base of natural logarithms). In applied (computational) mathematics, the form most used is,

$$y = 10 ^ x \quad \text{for all } x \qquad (01.18)$$

This is sometimes referred to as the "anti-logarithm" function.

The power function is defined to be,

$$y = x ^ a \quad \text{for all } x,a \qquad (01.19)$$

The most common occurances of the power function is the square (x^2) and the cube (x^3).

The most important thing to note about (01.18) and (01.19) is either the base (for the former) or the exponent (for the latter) is a constant. If we were to vary both the base and the exponent, we would have the "Coupled Exponent" function [01.01],

$$y = x ^ x = cxt(x) \qquad (01.20)$$

Other names for (01.20) are "Self-exponential" and "Second-order Towering exponent". In this work, we will used the term "coupled exponent".

The authors are unaware of any occurances of coupled exponents in nature; that is, there is no plant or animal that grows or multiplies in a coupled exponential fashion. The most common place to find (01.20) is in calculus textbooks where students are asked to calculate dy/dx via logarithmic differentiation. Advanced papers and tracts such as G. H.

Hardy's "Orders of Infinity" [01.02] and Paul Du Bois-Reymond's "Ueber asymptotischen Werte, infinitaere Approximationen und infinitaere Aufloesungen von Gleichungen" [01.03] use the coupled exponent for proving theorems involving asymptotic expensions but they do not address any of its special properties. These men took advantage of the super-fast increasing nature of the coupled exponent. Mr. Hardy in his "Orders of Infinity" introduces the notion of the "Tripled Exponent", that is,

$$y = x ^ x ^ x = x ^ {cxt(x)} \qquad (01.21)$$

and this is used to demonstrate convergence/divergence of series and for comparing one increasing function against another when their independent argument goes to infinity. We have the ordering,

$$x^{x^x} > x^x > x! > e^x > x^2 > x \quad \text{when } x \to \infty \qquad (01.22)$$

The following demonstrates the speed of x^x and x^x^x

x	x^x	x^x^x
1	1	1
2	4	16
3	27	7625597484987
4	256	1.340780792994E+154
5	3125	1.911012597945E+2184
6	46656	2.659119772153E+36305
7	823543	3.759823526784E+695974
8	16777216	6.014520753651E+15151335
9	387420489	4.281247731757E+369693099
10	1.0E+10	1.000000000000E+10000000000

(Fig. 01.02)

Coupled and Tripled exponents are special cases of what the American mathematician R.A. Knoebel calls "Hyperpowers". A hyperpower tells the number of times a number is exponentially iterated. Using the symbol "\" to denote the hyperpower operator, we have,

x^1 = x\1
x^x = x\2
x^x^x = x\3

To the best knowledge of the authors, no comprehensive theory has yet been developed; that is, hyperpowers have not been generalized to the point where one can compute z = x\y where x,y,z are complex numbers. We point this out to show that coupled and tripled exponents are part of something bigger.

BASIC PROPERTIES OF THE COUPLED EXPONENT

Equation (01.20) has some interesting properties. The first and second derivatives are,

$$\frac{dy}{dx} = x^x \cdot [1 + \ln(x)] \qquad (01.23)$$

$$\frac{d^2y}{dx^2} = x^x \cdot \left\{ [1 + \ln(x)]^2 + \frac{1}{x} \right\} \qquad (01.24)$$

The minimum of (01.20) is when (01.23) is equal to zero. This occurs at

$$x = 1/e = 0.3678794412$$

The limit at zero is,

$$\lim_{x \to 0+} x^x = 1 \qquad (01.25)$$

For arguments less than zero, the coupled exponent function is complex except when x is a negative integer. The value is then,

$$x^x = \frac{(-1)^x}{cxt(|x|)} \quad \text{for } x = -1,-2,-3,... \qquad (01.26)$$

When x is a non-integer and less than zero we obtain the result [01.04],

$$|x^x| = \frac{1}{cxt(|x|)} \quad \text{for all } x<0 \qquad (01.27)$$

For all x,y in R we have,

$$cxt(x*y) = [y^{(x-1)}*cxt(x)]^y*cxt(y) \qquad (01.28)$$

$$cxt(x^y) = cxt(x)^{[y*x^{(y-1)}]} \qquad (01.29)$$

For $f(x)=10^x$, we have the ratio,

$$\frac{f(x+1)}{f(x)} = 10 \qquad (01.30)$$

but for coupled exponents we get the following asymptotic expansion:

$$\frac{cxt(x+1)}{x^x} \sim e*x + \frac{e}{2} - \frac{e}{24x} + \frac{e}{48x^2} - \frac{73e}{5760x^3} + \frac{11e}{1280x^4} + ... \qquad (01.31)$$

This means that $cxt(x+1)/x^x$ can be approximated with the line

$$y = 2.7183 * x + 1.35914 \qquad (01.32)$$

One can expand the coupled exponent in a Taylor series around the point x=1,

$$cxt(1+x) = 1 + x + x^2 + \frac{x^3}{2} + \frac{x^4}{3} + ... \qquad (01.33)$$

This would be useful for series inversion near x=1.

The question is often asked: Can the integral of the coupled exponent be written in closed form? In terms of elementary functions, the answer is "no", but defining this integral to be a new higher function will prove useful in a later chapter.

The coupled exponent is an interesting higher function because both the base and exponent vary with respect to x, resulting in very rapid growth. There is much research that could be done on this function. Questions include: Could the coupled exponent serve as a basis for a new type of series? Could a new type of geometry based on R^R be developed? The problem of inverting the coupled exponent has lead to some interesting results. That is the focus of the rest of this book.

01.01:

One learns in school different methods of computing square roots. Methods date to before Christ. It was square roots that lead the Greeks to discover irrational numbers. Irrational numbers are those numbers that cannot be represented by a ratio of two integers. An example would be

SQRT(2) = 1.414213562...

The act of computing square roots is far more difficult than to compute the square. Most of the time, inverting a function is more difficult than the original function.

01.02:

One of the earliest computers, Konrad Zuse's Z1, was made from telephone relays. Konrad Zuse was Germany's leading computer scientist during WWII. He "invented" the computer because as a student of civil engineering, he was (according to an interview for PBS TV in the U.S.A.) "too lazy to perform the required calculations for bridge engineering". The Z1 made a great deal of noise due to the mechanical relays used. For programming, he used discarded film as punch tape to enter instructions into the machine. An assistant recommended that he use vacuum tubes instead of relays to speed the machine up 1000 fold. The Z1 could do a multiplication in 5 seconds. Had a vacuum tube equipped machine been built, its speed would be on a par with a modern programmable calculator.

The Americans built the ENIAC in 1946 which is acknowledged as the first electronic digital computer. This machine used vacuum tubes and was programmed by altering the wiring on the plugboard. It's main use was for computing ballistic tables for the U.S. Army and Navy.

Since the 1960's with the "new math" (an attempt by education "experts" to update mathematics education in the U.S.A.), students have been taught the basics of computer theory. The most basic is the binary number system which has two elements (0,1) where 0 is used to mean "off" and 1 "on". Computers represent all information as binary digits (known as "bits"). Sometimes computer scientists write binary numbers in base 8 or base 16 ("Octal" and "Hexadecimal"). This just amounts to grouping the bits into groups of 3 (for octal) or 4 (for Hexadecimal). The machines themselves work with bits in groups of 8 or 32. The grouping of 8 is called a "Byte" and is used to represent characters and numbers. The memory size of a machine is given (most of the time) in bytes. 1024 bytes (2^10) is called a Kilobyte and is noted by K such as 4096K. A Megabyte is 2^20 = 1048576 bytes and a gigabyte is 2^30 = 1073741824 bytes.
The group of 32 is called a "word" and is used for storing floating point numbers in the machine. On supercomputers like the Cray, a word is 64 bits long. Memory is measured in words instead of bytes. This is due to supercomputers being used mostly for calculation involving large amounts of numbers. Thus the memory size reported in words tells the user the number of floating point numbers that can be stored in the machine. Many problems in physics and engineering involve manipulating millions of numbers at one time.

The German mathematician Gottfried Von Leibnitz (~1670) thought binary numbers were the "natural" God-given number system because of its elegance. He also thought that someday court cases would be solved via calculations involving binary numbers to determine one's guilt or innocence. This would eliminate the need for lawyers, judges, long court cases and the expense they entail. Today, 300+ years later, we still have made no progress in this area.

01.03:

The first logarithm table was computed by John Napier of Scotland in 1604. He devised a method of computing logarithms to the base 'e' (2.718). The original use of his table was to aid in performing multiplication and division. The user would lookup the logarithm of the two numbers he wanted to multiply. He then added the two logarithms together and then looked in the table to see what number had the logarithm that equalled the computed sum.

In 1610, an Englishman, Henry Briggs, computed the first table of logarithms to the base 10. This was far more practical. Logarithms were used to compute tables of trignometric functions which were used for navigation. In the early 1600's England was in competition with Spain for land in the new world. In 1588, Spain tried to invade England because England broke from the Catholic Church and the Spanish hoped to get England back into the Catholic fold. At that time Spain had the best navigators in the world. The Spanish invasion failed due to bad weather and tough mobile ships piloted by Englishman who wanted nothing more to do with the Catholic Church (the Protestant Reformation had been ongoing for over 70 years). The logarithm table followed by the development of the slide-rule enabled England to advance past Spain in the field of navigation. By 1680, England was the world leader in science and mathematics.

These developments can be compared to our era of space competition between the U.S.A. and the U.S.S.R. where there was great focus on innovation. In the U.S.A. aerospace companies were awarded large contracts to develop smaller, faster computers, better space suits, etc. to aid the space effort.

Since 1610, many logarithm tables have been computed. Famous mathematicians such as Gauss and Schloemilch have computed high precision tables. Generations of high school students have had to learn linear interpolation ("reading between the lines" to obtain a value not in the table) while using 4 digit logarithm tables. The lucky ones got to use slide-rules.

Today, in our era of 'killer' Casio and 'hopped-up' HP calculators, there is little need for logarithm tables as these machines can compute logarithms to over 10 decimal places with the touch of a key. The basic properties of logarithms are still used however.

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page 161, Fig. 80

Chapter 02

Inversion of the Coupled Exponent

INTRODUCTION

The coupled exponent function is a monotonic increasing, infinitely differentialable continuous function for x>=1. Its increase is faster than any exponential with base a>1 as x goes to infinity. The question of inversion was first investigated by L. Euler in the late 1700's [02.01]. He was more concerned with towering exponents, that is the sequence,

$$x,\ x^x,\ x^{(x^x)},\ x^{[x^{(x^x)}]},\ \ldots \qquad (02.01)$$

He discovered that for x in [1/e^e,e^(1/e)] this sequence converged. (02.01) would converge to the solution of x=y^(1/y). For example: If we let x=0.5, which is within the domain [0.06598804, 1.44466786], and calculate the sequence (02.01) we find it converges to a number, whose value is y=0.6411857445. To check, we calculate y^(1/y) and find the result to be 0.50. This can be generalized as follows:

The "Coupled Root" is defined to be the inverse of x^x. That is,

$$y = x^x = \mathrm{cxt}(x) \qquad (02.02)$$

$$x = \mathrm{crt}(y) \qquad (02.03)$$

For y >= 1, the coupled root is single valued.
For y in [1/e^(1/e),1) the coupled root is multi-valued.
For y < 1/e^(1/e) the coupled root value is complex. (Fig. 02.01)

Euler's sequence is really the solution of the equation, x=y^(1/y) which can be solved in closed form.

$$x = \frac{1}{(1/y)^{(1/y)}} = \frac{1}{\mathrm{cxt}(1/y)}$$

$$\frac{1}{x} = \mathrm{cxt}(1/y)$$

$$y = \frac{1}{\mathrm{crt}(1/x)}$$

In modern language, Euler's sequence converges to 1/crt(1/x).

x=y^(1/y)	y=1/crt(1/x)
1.0E-1000	0.00258717431
1.0E-300	0.00715192375
1.0E-200	0.01000000000
1.0E-100	0.01755579499
1.0E-10	0.10000000000
1.0E-6	0.14152685655
1.0E-5	0.15946624592

1.0E-3	0.21951315163
0.01	0.27798742481
0.1	0.39901297826
0.25	0.50000000000
0.50	0.64118574451
1.00	1.00000000000
1.4	x1=1.8866633062, x2=4.4102927939
2.00	0.82467854614 - 1.5674321239 * i
3.00	0.22975010659 - 1.2664477436 * i
10.0	-0.11919307342 - 0.7505832939 * i
100.0	-0.17012713295 - 0.4239597520 * i
1000.0	-0.15749964580 - 0.2978178949 * i

(Fig. 02.01)

Towering exponents have been researched up to the present time. Papers by Woepcke [02.02], Knoebel [02.03] and others focus on the properties of towering exponents. The first author to concern himself with coupled roots is Gotthold Eisenstein. In his paper "Entwicklung von a^a^a^a...", [02.04], Eisenstein compiles the first known coupled root table for x=1,2,3,...40,50,60,99,100,101,...105 out to 7 significant figures. He comments that "this exercise [in calculating coupled roots] is instructive for the beginner in analytic geometry."

BASIC PROPERTIES OF THE COUPLED ROOT

Because the coupled root is the inverse of x^x, we should first compare coupled roots to logarithms. (Fig. 02.02)

$$\mathrm{crt}(x) > \log(x) \quad \text{for x in [1,1E+10)} \qquad (02.04)$$

$$\mathrm{crt}(x) = \log(x) \quad \text{at x = 1E+10} \qquad (02.05)$$

$$\mathrm{crt}(x) < \log(x) \quad \text{for x > 1E+10} \qquad (02.06)$$

x	log(x)	crt(x)
1	0.0	1.0
2	0.3010299957	1.559610469
3	0.4771212547	1.825455023
4	0.6020599913	2.0
5	0.6989700043	2.129372483
10	1.0	2.506184146
100	2.0	3.597285024
1000	3.0	4.555535705
1.0E+6	6.0	7.065796728
1.0E+10	10.0	10.0
1.0E+20	20.0	16.44640751
1.0E+100	100.0	56.96124843
1.0E+1000	1000.0	386.5220817

(Fig. 02.02)

As x-->infinity, the coupled root function goes to infinity at a slower rate than logarithms. That is,

$$\lim_{x \to \infty} \frac{\mathrm{crt}(x)}{\log(x)} = 0 \qquad (02.07)$$

To calculate the slope of the coupled root function we use,

$$\frac{d}{dx}\, crt(x) = \frac{1}{x*[1+\ln(crt(x))]} \qquad (02.08)$$

$$\frac{d^2}{dx^2}\, crt(x) = \frac{1}{x} * \frac{dy}{dx} - \frac{x}{y} * \left|\frac{dy}{dx}\right|^3 \quad \text{where } y=crt(x) \qquad (02.09)$$

The Taylor expansion around x=1 is,

$$crt(1+x) = 1 + x - x^2 + \frac{3}{2} x^3 - \frac{17}{6} x^4 + \frac{37}{6} x^5 + \ldots \qquad (02.10)$$

For v>0 and x in R we have,

$$\frac{cxt[v*crt(x)]}{x^v} = cxt(v)^{crt(x)} \qquad (02.11)$$

Can the integral of crt(x) we computed in terms of the elementary functions?
No, but the integral of crt(a^x) for a>0, can.

ORDERS OF FUNCTIONS AND THE COUPLED ROOT

In G.H. Hardy's "Orders of Infinity", Hardy uses an ordering scheme first devised by Du Bois Reymond for "sorting out" fast growing functions. The "Type" of a function is defined to be:

$$Typ[f(x)] = \frac{1}{f} * \frac{df}{dx} \qquad (02.11)$$

As examples, Typ(e^x) = 1, Typ(x^x) = 1+ln(x). From this, the fast growing functions can be "tamed". Hardy's system is based around e^x.

If we proposed the same type of system but based on coupled exponents, we would have,

$$T1[f(x)] = \frac{crt[f(x)]}{x} \qquad (02.12)$$

An example of this would be T1(10^x) = crt(10^x)/x.
The difficulty with this is so far, the coupled root cannot be asymptoticaly reduced to a "known" function. In the example cited, one does not yet know what the order of crt(10^x)/x is. We do know that it is less than 1 because $10^x < x^x$ for x-->infinity. It will be shown later that T1(10^x) ~ 1/log(x)

NUMERIC CALCULATION AND BIG NUMBERS

Coupled roots display "quasi-logarithmic" behavior. That is, coupled roots "almost" obey the basic laws of logarithms,

$$\log(x \cdot y) = \log(x) + \log(y) \qquad (02.13)$$

$$\log(x^y) = y \cdot \log(x) \qquad (02.14)$$

More important, as x --> infinity, can the coupled root be written in terms of elementary functions?

Attempts to solve this numerically lead to difficulties. Most small programmable calculators and pocket computers are limited to 10 decimal places and (more important) a dynamic range of 10^99. This means the largest coupled root one can compute is crt(9.999E99) = crt(10^100) which is just under 57. As it stands, it would be impossible to calculate say, crt(10^1000000) [value is 189481.3]. Asymptotics, as a branch of mathematics, is where one takes the "long range view". One tries to see what the function's behavior is as x-->infinity.

The big question is: Are coupled roots a new type of logarithm? Is there a duplication formula for the coupled root whereby when given a value for x and crt(x), can one calculate crt(2*x)?

We have raised more questions about coupled roots than answers. Coupled roots are a new type of higher function that has not been studied much by mathematicians. Can coupled roots be used in applications where sometype of "universal logarithmic" function is needed? This is an open area for research.

Note: We have not discussed much about tripled exponents. We can define the inverse of y=x^(x^x) to be x=trp(y) for all x. Tripled exponents and tripled roots will only be used for comparing fast growing functions. They too are an open topic of research.

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Chapter 03

Coupled Roots of Large Numbers

INTRODUCTION

Ask someone to solve y=x^x for x and chances are the first thing they will write is,

$$\log(y) = x * \log(x) \qquad (03.01)$$

and then,

$$x = \log(y)/\log(x) \qquad (03.02)$$

After a short time, they will conclude that the problem is unsolvable in closed form because they "cannot get rid of the x on the right hand side of the equation". They will be correct. The important thing to note is that the first step was taking logarithms of both sides in an attempt to clear the exponent (^) operation. The problem reduces to x times its logarithm. This looks a little less intimidating.

Have the same individual plot on an (x,y) graph the two functions,

$$y=crt(x) \quad \text{for } x>=1 \qquad (03.03)$$

-and-

$$y=\log(x) \quad \text{for } x>=1 \qquad (03.04)$$

It will appear that crt(x) > log(x) for all x. This is of course, wrong. A way is needed to "speed-up" the x value so we can plot for larger values of x.

Make the substitution, z = 10^x and plot now the functions,

$$y=crt(10^x) \quad \text{for } x>=0 \qquad (03.05)$$

$$y=x \quad \text{for } x>=0 \qquad (03.06)$$

Instead of two super-slowly increasing functions, one now sees a line at a 45 degree angle and a graceful curve starting at (0,1). Points can be read off the curve: (0.602, 2), (1.43, 3), (2.4, 4). The curve and line intersect at (10,10) thus driving home the fact that,

$$crt(x) = \log(x) \quad \text{at } x = 1E+10 \qquad (03.07)$$

Each step in the x direction now represents a decade (stepping thru a power of 10). At x=100, we have y = 56.96; at x=200, y = 100, etc.

WORKING WITH LARGE NUMBERS

The definition of "large number", for this discussion, is a value greater than 1.0E+100. This is beyond the range of most modern pocket calculators and pocket computers. Because early work with coupled roots was numeric based, and a need for handling large numbers arose, a new function call the "Wexzal" (corruption of the German word "Wurzel", meaning "root") was defined.

The wexzal is defined to be,

$$wzl(x) = crt(10^x) \qquad (03.08)$$

It is a function for x>=0; a double rooted relation
for x in [-log(e)/e,0). For x < -log(e)/e, the Wexzal is complex valued.

Now we have the means to calculate (on a standard calculator) coupled roots of up to 1.0 * 10^1.0E+99.
Example: wzl(1.0E+99) = crt[10^(1.0E+99)] = 1.030787889E+97

BASIC PROPERTIES OF THE WEXZAL FUNCTION

Some of the basic properties of wexzals include,

$$wzl(\log(x)) = crt(x) \qquad (03.09)$$

$$wzl(0) = 1 \qquad (03.10)$$

$$wzl(x) ^{\wedge} wzl(x) = 10^x \qquad (03.11)$$

$$y = wzl(x), \quad x = y * \log(y) \qquad (03.12)$$

$$wzl(x) * \log[wzl(x)] = x \; \text{-->} \; \log[wzl(x)] = x/wzl(x) \qquad (03.13)$$

The identity: log[wzl(x)] = x/wzl(x) , is very important in numeric calculation for one main reason: One can compute a logarithm (of a wexzal) by just dividing. Logarithmic calculation on computers and calculators is considered "expensive" in terms of CPU time. An example of this concern with "computer time" is in the field of realtime control. The computer must make calculations quickly enough so it can react to the inputs from the outside world (e.g. sensor on a manufactoring robot) in realtime. Another use for (03.13) is in the calculation of integrals involving the wexzal function.

Other identities include,

$$wzl(x*10^x) = 10^x \qquad (03.14)$$

$$\log\{cxt[wzl(x)^2]\} = 2 * x * wzl(x) \qquad (03.15)$$

$$sqr\{cxt[wzl(x)^2]\} = 10^{[x*wzl(x)]} \qquad (03.16)$$

The derivative of the wexzal can be computed as follows,

$$x = y * \log(y) \qquad (03.17)$$

$$\frac{dx}{dy} = m + \log(y) \quad \text{where } m = \log(e) \text{ [Log converion factor]} \qquad (03.18)$$

$$\frac{dy}{dx} = \frac{1}{m + \log[wzl(x)]} = \frac{1}{m + \dfrac{x}{wzl(x)}} \qquad (03.19)$$

The second derivative is,

$$\frac{d^2y}{dx^2} = -\frac{\frac{1}{y} - \frac{x\frac{dy}{dx}}{y^2}}{\left(\frac{x}{y} + m\right)} = -\frac{m}{y} * \left|\frac{dy}{dx}\right|^3 \qquad (03.20)$$

Further calculation of the derivative of y=wzl(x) leads to the taylor series around x=0.

$$wzl(x) = 1 + \frac{1}{m} x - \frac{1}{m^2*2} x^2 + \frac{4}{m^3*6} x^3 - \frac{27}{m^4*24} x^4 + \ldots \qquad (03.21)$$

Note that each term is in the form of

$$An = (-1)^{(n-1)} * \frac{cxt(n-1)}{m^n * n!} \quad \text{for } n=0,1,2,3,... \qquad (03.22)$$

Performing the ratio test leads to a radius of convergence of

$$|x| <= m/e = 0.1597680113 \qquad (03.23)$$

Reciprocating (03.21) leads to the series,

$$\frac{1}{wzl(x)} = 1 - \frac{1}{m} x + \frac{3}{m^2*2} x^2 - \frac{16}{m^3*6} x^3 + \frac{125}{m^4*24} x^4 + \ldots \qquad (03.24)$$

Where each derivative of 1/wzl(x) at x=0 is,

$$An = (-1)^n * \frac{(n+1)^{(n-1)}}{m^n} \qquad (03.25)$$

ASYMPTOTIC PROPERTIES OF THE WEXZAL

Asymptotics is the study of function behavior as x-->infinity. We say the f(x) is asymptotic to g(x) for x-->infinity when we have,

$$\lim_{x \to inf} \frac{f(x)}{g(x)} = 1 \qquad (03.26)$$

and it is written as f(x) ~ g(x). We shall use the notation f(1/x)~g(1/x) to mean,

$$\lim_{x \to inf} \frac{f(1/x)}{g(1/x)} = 1 \qquad (03.27)$$

The wexzal function is asymptotic to an expression involving logarithms. In Hardy's "Orders of Infinity", [03.01] he quotes a technique used by Du Bois Reymond in his "Infinitaercalcuel" for asymptotic solution of equations. We outline it here

Given the equation,

$$x = y * K(y) \qquad (03.28)$$

where $y^{(-v)} < K < y^n$ where v is "near" zero. If the increase of growth of K is slow enough where $K[y*K(y)]$ is like (in the asmptotic sense) $K(y)$ then we have,

$$y = x/K(y) \sim x/K(x) \qquad (03.29)$$

From this we can show that the wexzal is asymptotic to x/log(x).

THM: $wzl(x) \sim x/\log(x)$ (03.30)

Proof (via repeated use of L`Hospital's Rule):

$$\lim_{x\to\infty} \frac{wzl(x)*\log(x)}{x} = \lim_{x\to\infty} \left\{\frac{m*wzl(x)}{x} + \frac{\log(x)}{m+\log[wzl(x)]}\right\} \qquad (03.31)$$

Because wzl(x) < x for all x > 10, the term,

$$\lim_{x\to\infty} \frac{m*wzl(x)}{x} = 0 \qquad (03.32)$$

We now have,

$$\lim_{x\to\infty} \frac{\log(x)}{m+\log[wzl(x)]} = \lim_{x\to\infty} \frac{\frac{m}{x}}{\frac{m}{m*wzl(x)+x}} = \lim_{x\to\infty} [m*wzl(x)+x]/x$$

$$\lim_{x\to\infty} [m*wzl(x)+x]/x = \lim_{x\to\infty} \left[1 + \frac{m*wzl(x)}{x}\right] = 1 \qquad (03.33)$$

Therefore, (03.30) is true.

Let us see how the numbers compare,

x	wzl(x)	x/log(x)	ratio
100	56.96124843	50.0	1.139334969
1000	386.5220817	333.333333	1.159566245
1E+06	189481.2766	166666.667	1.136887659
1E+10	1105747503	1000000000	1.105747503
1E+50	2.069711620E+48	2.0E+48	1.034855810
1E+99	1.030787889E+97	1.01010101E+97	1.020480010

(Fig. 03.01)

Using (03.30) we can generate from eqn (03.19) the following result,

$$\frac{1}{m + \frac{x}{wzl(x)}} \sim \frac{wzl(x)}{x} \sim \frac{1}{\log(x)} \qquad (03.34)$$

This means,

$$\mathrm{wzl}(x+1) \sim \mathrm{wzl}(x) + \frac{1}{m + \dfrac{x}{\mathrm{wzl}(x)}} \sim \mathrm{wzl}(x) + \frac{1}{\log(x)} \qquad (03.35)$$

Using theorems from Du Bois Reymond's "Ueber asymptotische Werthe, infinitaere Approximationen und infinitaere Aufloesungen von Gleichungen" [03.02] we present the following results where y=wzl(x).

$$\mathrm{wzl}(x+1) \sim \mathrm{wzl}(x) * e^{\frac{1}{y}\frac{dy}{dx}} \qquad (03.36)$$

$$\mathrm{wzl}(2 * x) \sim \mathrm{wzl}(x) \,\hat{}\, \left[1 + \frac{\log(2)}{m + \dfrac{x}{\mathrm{wzl}(x)}}\right] \qquad (03.37)$$

$$\lim_{x \to \mathrm{inf}} \left[\mathrm{wzl}(x) \,\hat{}\, \frac{dy}{dx}\right] = 10 \qquad (03.38)$$

$$\mathrm{wzl}[(x*\log(x))^v] \sim x^v/v*\log(x)^{(v-1)} \quad \text{such that } v>0 \qquad (03.39)$$

$$\mathrm{wzl}[\mathrm{sqr}(x*\log(x))] \sim \mathrm{sqr}[\mathrm{wzl}(4*x)] \sim 2*\mathrm{sqr}[\mathrm{wzl}(x)] \qquad (03.40)$$

$$\mathrm{wzl}(x^v*10^x) \sim x^{(v-1)}*10^x \quad \text{for all } v \text{ in reals} \qquad (03.41)$$

$$\mathrm{wzl}(x)^2 \sim 2*\mathrm{wzl}[x*\mathrm{wzl}(x)] \qquad (03.42)$$

More results can be found in the appendix.

ASYMPTOTIC EXPRESSION FOR TRP(10^x)

It was shown that an asymptotic expression could be developed for crt(10^x). It is possible to do the same thing for trp(10^x). Because the tripled root function is slower than the coupled root, we would expect a result involving either double logarithms or coupled roots. First let us prove that

$$e*x^{(x+1)} \sim \mathrm{cxt}(x+1) \qquad (03.43)$$

Rewriting this as,

$$x^{(x+1)} \sim \mathrm{cxt}(x+1)/e \qquad (03.44)$$

and taking natural logarithms of both sides and dividing we get,

$$\lim_{x \to \mathrm{inf}} \frac{(x+1)*[\ln(x+1)-1]}{(x+1)*\ln(x)} = \lim_{x \to \mathrm{inf}} \frac{\ln(x+1)}{\ln(x)+1} = \lim_{x \to \mathrm{inf}} \frac{\ln(x)+1/x}{\ln(x)+1} = 1 \qquad (03.45)$$

Therefore e*x^(x+1) ~ cxt(x+1). We want to show that,

$$trp(10^x) \sim 1 + crt\left[\frac{x}{e*\log(x)}\right] \qquad (03.46)$$

Start by computing the inverse of both sides. For the left side we get,

$$x = y^y*\log(y) \qquad (03.47)$$

For the right side we have,

$$\frac{x}{\log(x)} = e * (y-1)^{(y-1)} \qquad (03.48)$$

The asymptotic solution of y=x/log(x) is

$$x \sim y*\log(x) \qquad (03.49)$$

So we get for the solution to (03.48) is

$$x \sim e*(y-1)^{(y-1)} * (y-1) * \log(y-1) = e*(y-1)^{y} * \log(y-1) \qquad (03.50)$$

From (03.43) we can deduce that

$$x^{(x+1)} \sim \frac{1}{e} * cxt(x+1) \qquad (03.51)$$

Letting A=y-1 in (03.50) we get,

$$x \sim e * A^{(A+1)} * \log(A) \sim (A+1)^{(A+1)} * \log(A) \qquad (03.52)$$

Which becomes,

$$x \sim y^{y} * \log(y-1) \qquad (03.53)$$

See if we can "dispose" of the (y-1) in the logarithmic term by taking limits.

$$\lim_{y\to\infty} \frac{y^y*\log(y-1)}{y^y*\log(y)} = \lim_{y\to\infty} \frac{\log(y-1)}{\log(y)} = \lim_{y\to\infty} \frac{\log(y)-m/y}{\log(y)} = 1 \qquad (03.54)$$

Therefore (03.46) is true.

The two asymptotic developments have defined how quickly the coupled root and tripled root increase as x increases. They can be written as,

$$crt(x) \sim \frac{\log(x)}{\log[\log(x)]} \qquad (03.55)$$

$$trp(x) \sim 1 + crt\left\{\frac{\log(x)}{\quad}\right\} \qquad (03.56)$$

e*log[log(x)]

Asymptotic expressions have been developed for both coupled and tripled root of 10^x. Both these functions will be used for solving equations involving logarithms in asymptotic and closed form.

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Chapter 04

Solution of Equations via Wexzals

INTRODUCTION

Coupled Roots (and Wexzals) give one the ability to solve various transcendential equations in closed form. By "closed form" we mean the ability to write the equation as a formula without the use of any infinite process such as integration or summation. The definition of closed form is also dependant on what functions are considered "elementary". Most mathematicians consider the trigonometric, logarithmic and hyperbolic functions to be elementry. A humorous definition of "elementary functions" is "what can be found on the face of a scientific calculator". These functions are studied in great detail by students of mathematics {04.01}. The so-called "higher functions" such as the Bessel, Gamma and Zeta functions are defined either in terms of a series or an integral. These functions got named and tabulated because they were used to solve important problems in physics and engineering [04.01]. Higher functions have specialized use and (sometimes) very interesting properties [04.02]. For the sake of numeric calculation, one can view these functions like the elementary functions. In this work, the Coupled Root and Wexzal are higher functions. If one accepts these higher functions (uses the notation for them; not write the series or integral representation), then one has expended ones ability to write equations (or their solution) in closed form.

For example, a differential equation might have the Bessel function as a solution. If you accept the Bessel function as a "basic" function then the differential equation's solution would be in closed form. If you do not accept the Bessel function as being "basic", then you would have to write the solution in series form and thus it would not be in closed form.
For this book, higher functions such as Bessel, Wexzal, etc. are considered "basic".

Why the obsession with closed form? The main advantage of writing the solution of transcendential equations in closed form is the ability to obtain numerical values to high precision quickly. Closed form results make the equations much easier to manipulate as no tests for series convergence need be made. Some of the equations that can be solved with the Wexzal are y=x+log(x), y=x*wzl(x), y=x^2+10^x, etc.

EQUATIONS INVOLVING LOGARITHMS/EXPONENTS

The classic equation that is studied in calculus is,

$$y = x^{(1/x)} \qquad (04.01)$$

The solution is (see chapter 2),

$$x = \frac{1}{\mathrm{crt}(1/y)} \qquad (04.02)$$

From this, we can solve,

```
          x
    y = ------                                                  (04.03)
        log(x)
```

This is done by,

```
    1   log(x)
    - = ------                                                  (04.04)
    y     x

    10^(1/y) = x^(1/x)                                          (04.05)

            1
    x = ---------                                               (04.06)
        wzl(-1/y)
```

From this, we can see that depending on the value of y, x can take either real or complex values. For y<0, x is single valued and real. For y>=e, x has two real values. For 0<=y<e, x is complex.

Another example is,

```
    y = x^2*log(x)                                              (04.07)

    2*y = 2*x^2*log(x) = x^2*log(x^2)                           (04.08)

    x = sqrt[wzl(2*y)]                                          (04.09)
```

Another example, this time with exponents, is,

```
    y = x+10^x                                                  (04.10)
    Let z=10^x so we get,
    y = log(z)+z                                                (04.11)
    10^y = z*10^z                                               (04.12)
    Let u=10^z
    10^y = u*log(u)                                             (04.13)
    u = wzl(10^y)                                               (04.14)
    z = log(u) = log[wzl(10^y)]                                 (04.15)
    x = log(z) = log{log[wzl(10^y)]}                            (04.16)
```

EQUATIONS INVOLVING WEXZALS

From the preceding section, it is clear that most logarithmic equations can be solved in closed form with Wexzals. If Wexzals are needed to solve logarithmic equations in closed form, then does it follow that something "higher" is required to solve Wexzalic equations in closed form? For most Wexzalic equations, the answer is "no". An example:

Solve,

```
    y = x*wzl(x)                                                (04.17)
    2*y = 2*x*wzl(x)                                            (04.18)
    2*y = 2*wzl(x)^2*x/wzl(x)                                   (04.19)
    2*y = wzl(x)^2*log[wzl(x)^2]                                (04.20)
    wzl(2*y) = wzl(x)^2                                         (04.21)
    sqrt[wzl(2*y)] = wzl(x)                                     (04.22)
    cxt{sqrt[wzl(2*y)]} = 10^x                                  (04.23)
    x = log(cxt{sqrt[wzl(2*y)]})                                (04.24)
```

All we did was take advantage of the identity property, log[wzl(u)]=u/wzl(u) and try to place the equation in a form of y=<term_of_x>*log[<term_of_x>] so we can then solve for x.

A more complex example would be solving y=x+wzl(x) as the addition would be expected to make the equation more difficult to solve. This is based on our experience with y=x+10^x from before.

Solve,

```
    y = x+wzl(x)                                            (04.25)
    10*y = 10*[x+wzl(x)]                                    (04.26)
    10*y = 10*{wzl(x)*[log(wzl(x))+1]}                      (04.27)
    10*y = 10*{wzl(x)*[x/wzl(x)+1]}                         (04.28)
    10*y = 10*wzl(x)*log[10*wzl(x)]                         (04.29)
    wzl(10*y) = 10*wzl(x)                                   (04.30)
    wzl(10*y)/10 = wzl(x)                                   (04.31)
    cxt[wzl(10*y)/10] = 10^x                                (04.32)
    x = log{cxt[wzl(10*y)/10]}                              (04.33)
```

An interesting "trick" discovered by one of the authors is:
Given,

```
    y = x+f(x)  whose solution is x=g(y)                    (04.34)
```

The solution of

```
    y = x+INVf(x)  where INVf(x) means the inverse of f(x)  (04.35)
```

is given by,

```
    x = f[g(y)]                                             (04.36)
```

An example of using this is the following:

```
    y = x^x*10^x                                            (04.37)
    log(y) = x+x*log(x)                                     (04.38)
    Let f(x) = wzl(x)  and using (04.24) we have
    g(y) = log{cxt[wzl(10*y)/10]}                           (04.39)
    INVf(x) = x*log(x) so
    INVf[g(y)] = wzl(10*y)/10                               (04.40)
    So the final answer to (04.20) is,
    x = wzl[10*log(y)]/10                                   (04.41)
```

LIMITATIONS

Is the Wexzal "all-powerful" in solving logarithmic equations? The answer is no. An example of an equation that (so far) has eluded solution is,

```
    y = 10^x*log(x)                                         (04.42)
```

which is the same form as,

```
    y = x + log[log(x)]                                     (04.43)
```

Note that (04.43) has a double logarithmic term and a linear term. When there is a linear term and either a logarithmic or exponential term in an equation, we call this a "logarithmic difference of 1". The logarithmic difference is defined to be the number of types a logarithm (or exponential) need be computed to transform the linear term into the logarithmic one. E.g.

```
    x+log(x)  ==> logarithmic difference of 1               (04.44)
    10^x*x    ==> logarithmic difference of 1               (04.45)
    10^x*log(x) ==> logarithmic difference of 2             (04.46)
    x+log[log(x)] ==>  logarithmic difference of 2          (04.47)
```

We think the Wexzal is incapable of solving equations with logarithmic differences greater than one. The theory for this needs to be further developed.

AN EQUATION INVOLVING TRIPLED ROOTS

In Chapter 03 we discussed the basics of Tripled Roots. One equation whose solution involves tripled roots is the following:

$$y = x*10^{x} + \log(x) \qquad (04.48)$$

By taking anti-logarithms of both sides twice, the solution becomes clear,

$$10^{y} = x*10^{x*10^{x}} \qquad (04.49)$$

$$10^{10^{y}} = (10^x)^{(10^x)^{(10^x)}} \qquad (04.50)$$

From this we get,

x = log{trp[10^(10^y)]} (04.51)

Which shows that the addition of a simple logarithmic term can force an equation to be unsolvable (in closed form) with Wexzals only. From this, it appears that Tripled Roots is a "higher" order function than Wexzals in terms of solving equations just as Wexzals are "higher" compared to logarithms when it comes to solving equations. The theory relating Wexzals and Tripled Roots needs further development. The only major results relating this theory is,

$$y = \frac{1}{x} \quad , \quad y = \log(x) \quad \text{meet at } x = 2.506184146 = crt(10) \qquad (04.52)$$

$$y = x^{x} \quad , \quad y = wzl(x) \quad \text{meet at } x = 1.923580364 = trp(10) \qquad (04.53)$$

Where the first pair of equations lead to a Coupled Root based constant and the second pair (also "simple" functions) lead to a Tripled Root constant.

CONCLUSION

Coupled and Tripled Roots enable one to solve equations in closed form that before were impossible. This gives one a better understanding of the nature of the solution of the equation. The major area of interest is the relationship (if there is any) between Coupled and Tripled Roots in terms of solving equations.

04.01:

Mathematical tables and handbooks have existed since before Henry Briggs published his logarithmic (base 10) tables in 1610. The first effort to compile an extensive handbook containing all of the important higher functions was by Professors Eugene Jahnke & Fritz Emde in Stuttgart Germany in 1909. In their Forword, Jahnke & Emde stated that they were printing their book with German text on one side of the page and English on the other side. This was to make the book accessible to English and American mathematicians as well. This was five years before the start of WWI and Europe was prosperous and peaceful. There was much interchange between German and English mathematicians. The center of English mathematics was Cambridge where G.H. Hardy was located. Goettingen was the center of German mathematics. This small quiet German Universitaetstadt was where the great Karl Friedrich Gauss worked. The Goettingen university was under David Hilbert who in 1900 proposed a set of problems that would take mathematicians 100 years to solve.

In 1933, Jahnke & Emde saw that their book was a "best seller" and released a second edition. They added some tables and fixed mistakes found in the first edition. In 1938, they released the third edition. Their last edition was in 1941 during WWII. In 1945 the American publisher Dover (known for re-printing scientific classics for low price) released the 1941 edition.

In 1954, during the Cold War, two American professors, Milton Abramowitz and Irene Stegun with support from the National Buro of Standards (now called ANSI) and MIT published "the mother of all handbooks" titled "Handbook of Mathematical Functions with Formulas, Graphs and Mathematical Tables". This large work was intended to be a larger, more accurate version of Jahnke & Emde's work. Abramowitz & Stegun used digital computers (early IBM mainframes) to generate their tables. They included functions most useful for applied science as scientific research (mostly in rockets and nuclear weapons) was going on at full speed due to the Cold War. The first edition, due to the size and scope, was riddled with mistakes. By 1971, there were 10 editions. Dover has reprinted the ninth edition in paperback.

Today (1998) small powerful computers have almost removed the need for handbooks like these. In 1954, computers were expensive and scarce; the average researcher had to use a slide rule and (if lucky) a mechanical calculator that could add, subtract, multiply and divide. Today's researcher can obtain a computer costing less than $3000 (5000DM). Such a machine can do over 25 million 15-digit calculations per second; outperforming a Cyber 7600 from the late 1960's.

References for Chapter #04

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G.E. Stechert, Germany, 1941

Chapter 05

Integrals Involving Wexzals

INTRODUCTION

The Calculus is one of mankind's greatest achievments. It enabled one to now solve dynamic problems (where there is a rate of change) instead of just classical static problems. The Calculus had been in the making since Johann Kepler computed volumes of wooden beer kegs via a numerical technique similar to Simpson's Rule. Sir Isaac Newton and Gottfried Von Leibniz are today considered co-inventors of the Calculus because they were the first to state that differentation (computing slopes) and integrating (computing infinite sums) are inverse operations. Since the 1670's, the theory behind the Calculus has been made more rigorous.

Calculus (as taught in U.S. universities) is very much like joining the army. First year (Freshman) Calculus is "boot camp" for mathematics and physics majors. In the army, the recruit is told by his drill sergeant, "Here is your uniform and rifle, recruit! Learn it, live it, love it! Now drop and give me 50 [pushups]!". The university student buys a 1000 page calculus text and before he realizes it, he is cranking out integrals at 3 AM in his dorm. Why the focus on computing integrals?

Integration is the act of finding the function that when differentated gives the original function back. This sought-after function is called the integral of the original function. The integral also gives a formula for the area under the curve of the original function. When one solves a differential equation (equation with derivatives in it), the last step is to compute an integral. So a first year student is really practicing the "end-game" of solving differential equations. In his second or third year of study, he will learn the theory of differential equations. Since he has had much practice in computing integrals, this need not be focused on.

There is one thing that is sometimes overlooked in all of this. Integrals of some functions cannot be computed in closed form. These integrals are given names and tabulated. The most famous of these are the Jacobi Elliptic integrals. These are used to compute the distance a planet travels in its orbit around the sun. If the orbit were a circle, then the calculation of the distance is just 2 * Pi * radius_from_sun. But because the orbit path is an ellipse, there is no closed form (with respect to standard functions) solution. These integrals got named (due to their importance) and tables of integrals involving these functions and values have been compiled also.

From the 1820's to 1900, a branch of mathematics called "Higher Analysis" was devoted to the topic of new integrals and their properties. Functions like the Gamma Function, Bessel Functions (all kinds and orders), Spence's Integral and others have been compiled. This activity reached a peak in the 1890's when it became almost a "sport" for mathematicians to add to the growing stockpile of integral formulae. The lists grew more baroque and exotic. Today, we have whole mathematical handbooks devoted to lists of integrals. [05.01] By 1914, between George Canter's Set Theory and David Hilbert of Goettingen call for mathematicians to solve theoretic problems involving the foundations of mathematics, interest in Higher Analysis tapered-off. Today, super fast computers can numerically solve differential equations that would have one of these exotic integrals as a solution. BFI {05.01} has replaced elegance.

When one learns the various techniques of integration, one learns that,

$$\int 10^x \, dx = \frac{1}{m} * 10^x + c \qquad (05.01)$$

and,

$$\int x^2 \, dx = \frac{x^3}{3} + c \qquad (05.02)$$

But what about,

$$\int x^x \, dx = ? \qquad (05.03)$$

Equation (05.03) cannot be computed in closed form. Neither can,

$$\int crt(x) \, dx \qquad (05.04)$$

Both of these do lead to a new higher unnamed function.

INTEGRALS INVOLVING THE WEXZAL FUNCTION

If we cannot (yet) compute the integral of the Coupled Root, can we nevertheless compute the integral of the Wexzal? Yes, it is surprisingly easy,

$$\int wzl(x) \, dx = \int y*[m+\log(y)] \, dy \quad \text{where } y=wzl(x) \qquad (05.05)$$

From this we get,

$$\int wzl(x) \, dx = \frac{1}{2} * x* wzl(x) + \frac{m}{4} * [wzl(x)-1] + c \qquad (05.06)$$

It was found that the integrals of x*wzl(x), 1/wzl(x), x/wzl(x), wzl(x)^2 and sqrt[wzl(x)] could be calculated in closed form using only the elementary functions and the Wexzal. The question became "Using just the elementary functions and the Wexzal, can any 'simple' Wexzalic expression be integrated in closed form?"

In April of 1983, the work was begin on a related question. Does the series,

inf

$$\sum_{k=1} \frac{1}{k \cdot wzl(k)} = ? \qquad (05.07)$$

converge? If answer this, we can use the "Integral Test" for series convergence. This involved computing the integral,

$$\int_{1}^{\infty} \frac{1}{x \cdot wzl(x)}\, dx = ? \qquad (05.08)$$

Using wzl(x)~x/log(x), we can see that this integral converges. The problem is "transformed" into,

$$\int_{1}^{\infty} \frac{\log(x)}{x \cdot x}\, dx < \infty \text{ because } \log(x)<x \text{ for } x \to \infty \qquad (05.09)$$

To compute this numerically on a PC-4 pocket computer (battery powered 1568 step BASIC programmed machine) would have been difficult as the problem stood due to the very slow convergence. To speed-up convergence, we re-write (05.08) as,

$$\int_{0}^{\infty} \frac{dx}{wzl(e^x)} = 0.6508866537... \qquad (05.10)$$

We then use Simpson's Rule with increasing number of intervals to obtain the approximation given.
    Another related problem was the following: The integral of d[wzl(x)]/dx is just wzl(x). It can be shown (see Chapter 06) that,

$$\frac{d}{dx} wzl(x) = \frac{1}{m + \frac{x}{wzl(x)}} \sim \frac{wzl(x)}{x} \sim \frac{1}{\log(x)} \qquad (05.11)$$

What is the integral of wzl(x)/x? The last expression in (05.11) suggests the Exponential Integral. This is a function that is tabulated in the Abramowitz & Stegun Handbook of Mathematical Functions. Jahnke & Emde's Handbuch contain it also. To compute the integral of wzl(x)/x we do the following,

$$\int \frac{wzl(x)}{x}\, dx = \int \frac{y}{y \cdot \log(y)} \cdot [m + \log(y)]\, dy \quad \text{where } y=wzl(x) \qquad (05.12)$$

```
    /                 /
    | m + log(y)      |   m
    | ---------- dy = | ------ dy + y                          (05.13)
```

```
    |   log(y)        | log(y)
    /                 /

    /             /
    |   m         |  dy
    | ------ dy = | ----- = Ei[ln(y)] + c                       (05.14)
    | log(y)      | ln(y)
    /             /
```

So we have as final result,

```
    /
    | wzl(x)
    | ------ dx = wzl(x) + Ei{ln[wzl(x)]} + c                   (05.15)
    |   x
    /
```

Following the same idea, (05.08) can be computed in closed form also. It is,

```
    /
    |    dx        1
    | -------- = ------ - Ei{-ln[wzl(x)]} + c                    (05.16)
    | x*wzl(x)   wzl(x)
    /
```

It appears that the Exponential Integral "fills-out" the list of Wexzalic integrals that can be computed in closed form. It is most useful for expressions involving reciprocals. An example will make this clear.

```
    /
    |     1       1        1                   1
    | wzl(-) dx = - * {---------- - ln[ln[wzl(-)]]} + c         (05.17)
    |     x       m          1                 x
    /                  ln[wzl(-)]
                             x
```

But...

```
    /
    |   dx       x      1                1
    | ------ = ------ + - * Ei{-2*ln[wzl(-)]} + c               (05.18)
    |     1        1    m                x
    / wzl(-)   wzl(-)
          x        x
```

Which contains the Exponential Integral. Equation (05.15) is really the integral of 1/log[wzl(x)]. Can this help in computing integrals involving Coupled Roots?

INTEGRALS INVOLVING COUPLED ROOTS

There is no (known) way to compute the integral of Coupled Roots without defining a new function that is the integral of the Coupled Root. However, some Coupled root related expressions can be integrated in closed form.

```
    /
    |     dx       -ln[crt(x)]-2
    | ---------- = ------------- + c                            (05.19)
    | x*crt(x)^2       crt(x)
    /
```

By inspection we can show that,

$$\int_{1}^{inf} \frac{dx}{x*crt(x)^2} = 2 \qquad (05.20)$$

Another example is,

$$\int \frac{dx}{x*crt(x)} = \frac{ln(x)}{crt(x)} * \left[1 + \frac{ln(x)}{2*crt(x)}\right] + c \qquad (05.21)$$

Since Coupled Roots are slower growing than logarithms, it would be of interest to compute,

$$\int_{1}^{inf} \frac{crt(x)}{x^2} dx = \int_{0}^{1} crt\left(\frac{1}{x}\right) dx = 1 + \int_{1}^{inf} \frac{dx}{x^x} = 1.7041699552... \qquad (05.22)$$

Which is expected considering that,

$$\int_{1}^{inf} \frac{dx}{x^2} = 1 \qquad (05.23)$$

In order to compute the integrals of x^x, $1/x^x$, crt(x) and 1/crt(x) four new functions need to be defined.

INTEGRALS THAT CANNOT BE WRITTEN IN CLOSED FORM

There are integrals involving Wexzals and Coupled Roots that have, so far, resisted solution. Amoung them are, x/wzl(e^x) [used in gunpowder pressure curve research], wzl(x)*wzl(1/x) [theoretic interest in product of two terms one of which contain a reciprocal], log(x)*wzl(x), wzl(x)/e^(k*x) [used for LaPlace transforms] and others. The small collection given in the appendix is just a start in an area that requires more research.

CONCLUSION

Equations involving Wexzals are, in general, easier to integrate than those involving coupled roots. It was believed that the Exponential Integral could be used to aid in integrating all Wexzalic equations. This appears to not be true as there are integrals that (so far) cannot be integrated in closed form. Almost no research has been done on combining trig functions with the Wexzal function (with an eye towards FFT's and vibration theory).

05.01:
BFI means "Brute Force and Ignorance". This is not meant in a negative way but refers to taking advantage of the speed of computers to solve

mathematical problems. The best example is the "Traveling Salesman Problem" where every path is tested for least cost. It would be better to derive the optimal solution instead of having a computer try every single combination/path to find the lowest cost. Sometimes the derivation is not possible and/or the derivation would take longer than it would to just have the computer find the solution. Many mathematicians look upon this method of solving problems as "uncouth".

Most scientific research centres that perform this kind of work use the most powerful computers available such as Cray Y-MP/C-90's, Convex C-3800's and NEC SX-3. Unlike "pure" mathematicans, applied mathematicans are more interested in the practical solution to problems.

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Chapter 06

Asymptotics Involving Wexzals

INTRODUCTION

Asymptotics concerns itself with describing functions as their argument goes to infinity. Most of the time, this helps to simplify calculation and analysis as minor terms in the expansion can be ignored. For two functions, f(x) and g(x) we say f(x) is asymptotic to g(x) when,

$$\lim_{x \to \infty} \frac{f(x)}{g(x)} = 1 \qquad (06.01)$$

This is written as,

$$f(x) \sim g(x) \qquad (06.02)$$

A simple example is,

$$x^2 + x \sim x^2 \qquad (06.03)$$

What this says is "for very large x, the linear term (x) is overshadowed by the x^2 term. Also $(x^2+x)/x^2 = 1+1/x$ and as x-->infinity, the 1/x term disappears. Therefore $x^2+x \sim x^2$".

We can also use this notation to describe a function about zero by using 1/x instead of x in (06.02). By using classical MacClaurin series on 1/wzl(x) we obtain,

$$\frac{1}{\mathrm{wzl}\left(\frac{1}{x}\right)} \sim 1 - \frac{1}{m*x} + \frac{3}{2*m^2*x^2} - \frac{8}{3*m^3*x^3} + \ldots \qquad (06.04)$$

The German mathematician, Du Bois Reymond [06.01], in 1874 wrote a paper, "On Asymptotic values, Infinite Approximation and Resolution of Equations" outlining theorems that apply for different types of equations. In 1910, G.H. Hardy in his "Orders of Infinity" [06.02] further expand on this topic {06.01}. Hardy's main type of function was one that had the following property:

$$\frac{df(x)}{dx} \sim \frac{f(x)}{x} \qquad (06.05)$$

The Wexzal obeys (06.05),

$$\frac{d}{dx}\mathrm{wzl}(x) = \frac{1}{m + \frac{x}{\mathrm{wzl}(x)}} \sim \frac{\mathrm{wzl}(x)}{x} \qquad (06.06)$$

In chapter #03, it was shown that,

$$wzl(x) \sim \frac{x}{\log(x)} \qquad (06.07)$$

This says the Wexzal really boils down to something simple as x goes without bounds. Unlike most trig or logarithmic functions, there is no simple known addition theorem for the Wexzal. That is, we do not know what the form of g(x) and h(x) would be in,

$$wzl(a+b) = g(a) + h(b) \qquad (06.08)$$

but there is an asymtotic version that warrents attention.

ADDITION THM

For a function f(x) such that,

$$\lim_{x \to \infty} \frac{f(x)}{x} = 0 \qquad (06.09)$$

and,

$$f(x2) > f(x1) \text{ when } x2>x1 \qquad (06.10)$$

we have,

$$wzl[x+f(x)] \sim wzl(x) + \frac{f(x)}{\log(x)} \qquad (06.11)$$

Proof:
Using (06.07) we have,

$$wzl[x+f(x)] \sim \frac{x+f(x)}{\log[x+f(x)]} \sim \frac{x+f(x)}{\log[x*(1+f(x)/x)]}$$

We said that f(x)/x->0 so,

$$wzl[x+f(x)] \sim \frac{x+f(x)}{\log(x)+\log[1+f(x)/x]} \sim \frac{x+f(x)}{\log(x)} = \frac{x}{\log(x)} + \frac{f(x)}{\log(x)}$$

Therefore,

$$wzl[x+f(x)] \sim wzl(x) + \frac{f(x)}{\log(x)} \qquad (06.12)$$

Let's test this: For f(x)=1 we have,

$$wzl(x+1) \sim wzl(x) + \frac{1}{\log(x)} \qquad (06.13)$$

Which is in agreement with (06.06) in that,

$$\frac{wzl(x)}{x} \sim \frac{x}{x*\log(x)} = \frac{1}{\log(x)} \qquad (06.14)$$

For an equation like wzl(x^2+x) we make the substitution z=x^2 and would expand wzl[z+sqrt(z)] thus obtaining,

```
                            x
    wzl(x^2+x) ~ wzl(x^2) + --------                        (06.15)
                            2*log(x)
```

WEXZALIC SHIFTING THM

    For a function f(x) that is monotonic increasing (as x gets larger, so does f(x)), one expects,

```
    crt[f(x)] < f(x) for all x>1                            (06.16)
```

What happens when one computes crt{wzl[f(x)]} instead of just crt[f(x)]?
We expect

```
    crt{wzl[f(x)]} < crt[f(x)] as x-->infinity              (06.17)
```

The question is "How much less? It is measurable?" Let y=f(x) and z=log(y).
We have by use of the addition theorem:

```
                        y
    crt{wzl[y]} ~ crt[------] = wzl{log(y)-log[log(y)]} = wzl[z-log(z)]
                      log(y)

    crt{wzl[y]} ~ wzl[z-log(z)] ~ wzl(z) - 1 = crt(y) - 1
```

Therefore,

```
    crt{wzl[f(x)]} ~ crt[f(x)] - 1                          (06.18)
```

This says that "wrapping" a Wexzal around f(x) before taking the coupled root just decreases the value of the coupled root by one.

INVERSE FACTORIAL EXPANSION

    Chapter 04 was about solving equations in closed form. One equation that cannot be solved in closed form is the Factorial function. The Factorial function is defined for integers as,

```
    n! = 1 * 2 * 3 * 4 ... n                                (06.19)
```

For non-integer arguments, the Gamma function is used. For large x Stirling's approximation is used,

```
              x   -x
    y = x! ~ x * e  * sqrt(2*Pi*x)                          (06.20)
```

The fact that Stirling's formula has a coupled exponent in it gives a clue on how one might be able to get an asymptotic expansion for the inverse factorial function.

The first thing we see is the term "x^x*sqrt(x)" which could make a problem in that we do not have a means to solve this exactly. We take the guess that

```
                          x
    cxt(x+1/2)/sqrt(e) ~ x * sqrt(x)                        (06.21)
```

We check this by computing the logarithm base e of the left hand side and getting,

$$(x+0.5)*\ln(x+0.5)-0.5 \qquad (06.22)$$

We know,

$$\ln(x+k) \sim \ln(x) + \frac{k}{x} \quad \text{for fixed } k \qquad (06.23)$$

So using (06.22) and (06.23) we get,

$$(x+0.5)*\left[\ln(x) + \frac{0.5}{x}\right] - 0.5 = x*\ln(x)+0.5+0.5*\ln(x)+ \frac{0.25}{x} - 0.5$$

Which when x-->infinity, reduces to

$$x*\ln(x)+0.5*\ln(x) \qquad (06.24)$$

This becomes,

$$x^x*\sqrt{x} \qquad (06.25)$$

Therefore (06.21) is true.

Now, we take Stirling's formula and after moving sqrt(2*Pi) to the left hand side and using (06.21) we obtain,

$$\frac{y}{\sqrt{2*\pi}} \sim \mathrm{cxt}(x+0.5)/\sqrt{e}*\exp(-x)$$

Moving sqrt(e), which is just a constant, to the left hand side, we have

$$\frac{y*\sqrt{e}}{\sqrt{2*\pi}} \sim \mathrm{cxt}(x+0.5)*\exp(-x)$$

Let z=x+0.5 so we can "clean-up" the 0.5 constant term

$$\frac{y*\sqrt{e}}{\sqrt{2*\pi}} \sim \mathrm{cxt}(z)*\exp[-(z-0.5)] = \mathrm{cxt}(z)*\exp(-z)*\sqrt{e}$$

Dividing both sides by sqrt(e) causes that term to disappear.

$$\frac{y}{\sqrt{2*\pi}} \sim \mathrm{cxt}(z)*\exp(-z) = (z/e)^z$$

Raise both sides to 1/e power so we can get a coupled exponent term on both sides.

$$\left|\frac{y}{\sqrt{2*\pi}}\right| \hat{}\ \left(\frac{1}{e}\right) = \mathrm{cxt}(z/e)$$

Take coupled root of both sides then multiply by e to get,

$$z \sim e \cdot \mathrm{crt}\{[y/\mathrm{sqrt}(2 \cdot Pi)]^{(1/e)}\}$$

But... x=z-0.5 so we obtain the final result:

$$x \sim e \cdot \mathrm{crt}\{[y/\mathrm{sqrt}(2 \cdot Pi)]^{(1/e)}\} - 0.5 \qquad (06.26)$$

Let us try a few numbers:

X	Inverse Factorial
6	2.990531111
24	3.993858573
120	4.995563516
3628800	9.998313202
20!	19.99932716
50!	49.99978963
10^100	69.95743568
10^1000	449.9099614
10^1000000	205022.1719

Fig 06.01

Figure 06.01 shows that (06.26) gets a smaller relative error as the argument increases.

Following the same type of process as before, one can obtain asymptotic solutions to equations such as,

$$y = \frac{(2 \cdot x)!}{x!} \qquad (06.27)$$

The solution is,

$$x \sim \frac{e}{4} \cdot \mathrm{crt}\{[y/\mathrm{sqrt}(2)]^{(4/e)}\} \qquad (06.28)$$

ASYMPTOTICS INVOLVING COUPLED EXPONENTS

An interesting result that is somewhat unexpected is the asymptotic expansion of,

$$y = \frac{\mathrm{cxt}(x+1)}{\mathrm{cxt}(x)} \qquad (06.29)$$

Being a faster increasing function than the exponential we expect the ratio to be greater than a constant. For the exponential (base 10) we have,

$$y = \frac{10^{x+1}}{10^{x}} = 10 \qquad (06.30)$$

Taking the natural logarithm of (06.29) and expanding yields,

$$\ln(y) = (x+1) * \ln(x+1) - x*\ln(x) \qquad (06.31)$$

Using the fact that,

$$\ln(x+1) \sim \ln(x) + \frac{1}{x} \qquad (06.32)$$

we obtain,

$$\ln(y) \sim (x+1)*[\ln(x) + \frac{1}{x}] - x*\ln(x) \sim \ln(x) + 1 + \frac{1}{x} \qquad (06.33)$$

We "ignore" the 1/x term and have,

$$\ln(y) \sim 1 + \ln(x) \qquad (06.34)$$

Which means the final result (first order term only) is,

$$y \sim e*x \qquad (06.35)$$

Further refinement on this leads to,

$$y \sim e*x + \frac{e}{2} - \frac{e}{24*x} + \ldots \qquad (06.36)$$

ASYMPTOTIC EXPANSION INVOLVING AN INTEGRAL

In chapter 05 we saw that the integral of 1/wzl(1/x) involved the Exponential Integral. It is,

$$S(x) = \int_0^x \frac{du}{\mathrm{wzl}(1/u)} = \frac{x}{\mathrm{wzl}(1/x)} + \frac{1}{m} * \mathrm{ei}\{-2*\ln[\mathrm{wzl}(1/x)]\} \qquad (06.37)$$

We know that,

$$\frac{1}{\mathrm{wzl}(\frac{1}{x})} \sim 1 - \frac{1}{m*x} + \frac{3}{2*(m*x)^2} + \ldots \qquad (06.38)$$

By integrating term by term in (06.38) we can say as a "first cut" the asymptotic expansion of (06.37) would be,

$$S(x) \sim x - \frac{\ln(x)}{m} - \frac{3/2}{m^2*x} + \ldots \qquad (06.39)$$

By using the right hand side of (06.37) along with the asymptotic expansion of Ei(x) one can obtain the following slightly more refined result:

$$S(x) \sim x - \frac{\ln(x)}{m} + \frac{1}{m} * (\gamma + \ln(2/m) - 1) - \frac{3/2}{m^2*x} + \ldots \qquad (06.40)$$

where "gamma" is the Euler Constant and is = 0.5772156649...

The only difference between (06.39) and (06.40) is the constant term. This constant term is about = 2.542964134

Why is this integral so important? It is used in the automobile acceleration model (Chapter 13). The asymptotic expansion makes it easier to obtain approximate answers when one does not have access to either a table or programmable calculator.

CONCLUSION

Some of the asymptotic properties of the Wexzal function were presented. These enable the researcher to obtain a "long-range" view of the behavior of the Wexzal. The Wexzal does not (unlike other functions) have "friendly" duplication formulae that make it easy to obtain numeric results. It does however, have asymptotic properties that are distinctive; the most noteworthy being the inverse factorial function. The authors are unaware of any application this might have but they believe it could serve in computer science (algorithm complexity theory) or statistics.

06.01:

G. H. Hardy was England's top mathematician in the beginning of the 20th century. He lectured at Cambridge and Oxford University. He was a pure mathematician who had no interest in applications but prefered to work in number theory and other theoretic areas.

He was a bit of a nationalist who wanted to improve the teaching of mathematics in England. Since the Newton/Leibniz dispute of the 1680's the English tended to stay with Newtonian notation and standards while the rest of the world moved ahead with the Leibnitzian system which while invented later was better.

In the 1670's Sir Isaac Newton of England and Gottfried Von Leibniz of Germany "invented" Calculus. The calculus was in "the making" for sometime but the two men were the first to pull all of the needed theory together to make it a unified system. Newton called his "Fluxions" and used it to solve planet orbit problems. He published first and used the well-known "dot" notation to denote time derivatives. Newton's main focus was to solve physics problems. Von Leibniz was a philopher and natural scientist who was interested in many fields including politics. He wanted to solve physics problems also. Leibniz invented the well-known "dy/dx" notation to better show that a derivative was a ratio. His notation was more powerful in that new propeties of derivatives can be discovered just by "playing" with the notation. Leibniz publish in the early 1680's and Newton accused him of "stealing" Newton's work. That is when the problems started. Newton was a scientific "super-star" by this time and his opinion was law (in England). He was also known to be sometimes "loud" and arrogant. Leibniz was much calmer and tried not to get caught up in the dispute. Mainland Europe (France, Saxony, etc.) saw the superiority of the Leibniz notation and started using it. During this time, the French and England were not on friendly terms. They competed for empire in the New World.

By 1890, the "place to be and be seen" in the mathematics world was Goettingen, in Lower Saxony (Niedersachsen) in the heart of Germany. Germany had unified in 1871 under Kronprinz Otto Von Bismarck. Germany now had an empire (Deutsches Reich), a Kaiser and the best scientific/mathematical establishment on the planet. Germany was now becoming Europe's new "superpower". The 1890's was a very nationalistic time and prestige meant everything. The English had their world-wide empire. The Americans sent steam ships to Japan to demonstrate Western technology while the Germans were busy building-up their industries. Even popular music was involved. The American composer John Philip Sousa and the German composer Karl Teike wrote marches reflecting the era and the glories of their countries. Today we hear these marches during national holidays such as 4th of July.

From the time of Fredrick the Great (Friedrich der Grosse) of Prussia, Germany has always had a strong university tradition. Goettingen was founded in 1737 by an English king, King George of England & Hannover as Lower Saxony (capitol Hannover) was under the English crown. When Lower Saxony came under Prussian rule, it was subjected to the same Prussian ideas of efficiency and Ordnung along with everything else. The German mathematician Felix Klein (famed for the Klein bottle) was a diplomat and was able to get the American banker/baron J. Rockerfeller to invest in Goettingen. This enabled the university to upgrade the library and to set-up a physics institute as well. By the 1890's Goettingen attracted world-wide attention as being the center of Western mathematics. It enjoyed this reputation until the end of WWII. By that time most of the mathematicians left for England or the U.S.A. to aid in America's war effort.

In 1987 one of the authors visited Goettingen to do research there and to see the historic sights. Goettingen is a university town with about 110,000 people. Unlike American universities (campuses) the "university" is spread out over the entire town. The Physics building is at one part of town; the mathematics building at another location. Students travel from building to building by 3-speed bicycle on the Berliner-Strasse. The house where Otto Von Bismarck studied law is still standing. There is a memorial to Karl Friedrich Gauss and a beerhall named after him. Today, we would say Gauss is a "local hero". Since 1990, Gauss is featured on 20-Deutschmark bills.

There is also a memorial to the solders killed in WWI. One can almost hear the sound of marching solders in time to Teike's "Alte Kameraden" played by a brass band as the Field-Gray spiked-helmeted Reichwehr marches off to the Front fuer Kaiser und Vaterland.

In the mathematics building on the second floor is a display of old slide-rules and other computing devices dating from Leibniz time. The main floor has a mathematical library with books from as far back as the 1700's to today. There is also a large collection of mathematical journals from around the world. A large percentage of the collection is composed of journals from the American Mathematical Association (AMA).

DuBois Reymond's paper was found in Crelle's Journal. Crelle's Journal is regarded by many to be "The Journal" to be published in due to Crelle's very high standards. Hardy's "Orders of Infinity" was found amoung the textbooks. Hardy's book presents the theory of asymptotics based on the concept of "known" functions. Known functions are those used as "benchmarks" to compare the new unknown function in terms of rate of growth. The exponential function, exp(x), is considered to be known. The power functions, x^n, are also known. Based on this, one can determine the basic asymptotic properties of an unknown function. Hardy "cleaned-up" Reymond's work in instead of defining the "type" of a function to be,

$$\text{type}[f(x)] = \frac{f(x)}{\dfrac{df(x)}{dx}}$$

Hardy changed it to be,

$$\text{type}[f(x)] = \frac{\dfrac{df(x)}{dx}}{f(x)}$$

and presented further development of the theory along with some applications. It is a "dense" book but careful reading leads one to most interesting results.

References for Chapter #06

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Chapter 07

Numerical Calculations & Computing Devices

INTRODUCTION

Up to this point, we have discussed the theoretic aspects of Coupled Roots and Wexzals. Included are integrals, solution of logarithmic equations and the main asymptotic property. This is fine for a foundation into the theory, but initial Wexzal results came from numerical research. The first part of this chapter outlines the rise of the calculator and microcomputer and the role they played in early Wexzal research.

CALCULATORS

In 1975, pocket calculators had just dropped in price to the point where they caught the public's attention. A TI SR-50A was $300 (US) and it was one of the best scientific models at the time. Hewlett Packard (HP) unvailed the HP-67 which was a scientific programmable pocket calculator which cost over $700 (US). This was, in essence, the first pocket "computer" in that the HP-67 could perform looping and branching (repeat a set of steps over & over and jump to different parts of the program) like a "real" mainframe computer. It used the famed RPN {07.01} system unlike the TI, Sharp and Casio which used AOS. Scientific calculators used scientific notation and displayed results out to 10 decimal places. By the end of 1975, slide rules (the unoffical symbol of engineering and science) was delegated to the museum. The 3-decimal-place analog irovy coated "slip-stick" just could not compete against the 1970's digital "Wundermaschine".

Small calculators, which could only add, subtract, multiply and divide cost from $10 to $50 depending on the model. These small machines have come to be known as "4-bangers" because of their limited abilities. They did not have scientific notation but could display answers out to 8 decimal places. There were many different brands such as Bomar (the Bomar Brain), Lloyds, Unisonic and others. These low cost machines launched a public debate centering on allowing students to use calculators in schools. This ranged from grade school to university. Many thought that students would be over dependant on the machines. Others stated that calculators were the wave of the future and anyone who did not know how to operate one would be "left behind".

Most calculators, up to 1977, used LED (Light Emitting Diodes) that displayed numbers in a bright, fire-like, red display (TI & HP). Other models had bright "Kelly-Green" displays (Sharp, Unisonic). So-called Nixie Displays were used in older desk-top models like accountants would use. These numbers appeared more rounded and easier to read than the well known 7 segment displays used in LED models. In 1976, the first LCD (Liquid Crystle Displays) appeared. They have a silver, liquid, placid appearance that can be read in day-light as they worked by reflected light instead of emitted light (LED). LCD were a boon for another reason: They expanded battery life greatly as the bulk of battery power was devoted to powering the LED display. A set of AA batteries would die within two hours of heavy use. With the new LCD, it was possible to build credit-card size machines that used "watch" batteries.

By 1979, the market "shake-out" (where the little companies get killed off and only the "big-boys" remain) was complete. The "big-4" are Texas instruments (TI), HP, Sharp and Casio. For $100 (US) one could obtain a Casio Fx-501P programmable calculator with LCD display and CMOS memory. This CMOS technology was a new low-power chip technology that enabled the machine to "remember" the program even when turned-off. Older models, when turned-off, would "forget" the program. These machines used a magnetic strip that was the size of chewing-gum for storing programs.

In September 1980, the first pocket computers appeared. These machines looked like calculators except they had tiny QWERTY keyboards and used BASIC as the programming language. They featured 12-character LCD displays and memories as large as 2K bytes. These machines had 1/4 the power of the early home computers (Apple II, CBM PET, etc) and yet were battery powered. The best known machines were the Sharp PC-1211 and PC-1500, Casio fx-702P and fx-700P (a.k.a. Radio Shack PC-4). Their main advantage over programmable calculators was that BASIC was used which meant a (somewhat) standard language can be used. Pocket computer memories were (compared to programmable calculators) very large. The main drawback to pocket computers is that they do not contain as many built-in functions as programmable calculators. Most of the early machines did not have the factorial or matrix functions built-in. A skillful user would have no problem programming these in. As recent as 1991, these machines appeared to be more popular in Europe then in the U.S.A. One of the authors, while on holiday in Munich, Germany noted that many computer/office-supply shops sold these machines. University students are largest users of pocket computers due to their low cost and ease of use. By 1995, low-end laptop PC-based computers have more-or-less polished off these machines.

Machines got smaller, faster and "smarter". By 1986, graphics calculators appeared. They use a dot-matrix display that looks like a small TV screen. These machines can plot graphs of functions and draw pictures. Casio, HP and TI are the leaders with these machines. They have the memory, built-in programs and speed to rivel early mini-computers of the late-1960's. Today, for $100 (US) one can buy a machine with the following features:

- > 32K (bytes) memory
- Communicate with PC type computer via cable
- Run for over 200 hours on a set of watch batteries
- Graphic displays (96 x 64 pixels)
- > 20 different program areas
- Ability to perform (along with the standard scientific functions),
    - Matrix operations (Inversion, Determinant, etc)
    - Complex Numbers
    - Statistics (Mean, Std. Deviation, Linear Regression)
    - Coordinate transformations
    - Differential Equations & Integration (Runge Kutta/Simpson)
    - Programmable with >10 levels subroutine levels, etc.

This sounds much like a well equipped PDP-11/04 from early 1970!

Calculators have certain features that are different from most computers. Calculators perform their calculations in Binary Coded Decimal (BCD) where each digit is represented by 4 bits. This makes for a more complex circuit set as the machine needs to perform special bit manipulations instead of performing the calculation in floating point binary. BCD has the advantage of maintaining precision as there is no loss due to converting to/from decimal. Most calculators have a dynamic range of 10^(+-99) instead of some value that is a power of two like 1.7*10^38 Scientific calculators employ what is known as "Guard digits". Guard digits are extra digits that are used in a calculation to maintain precision. The result is rounded to the final answer and then presented to the user. The Casio Fx series use 13 decimal digits for all calculations. The user only sees 10. The 3 guard digits are used to protect against rounding errors. The user can assume that (given a non ill-conditioned sequence of calculation) the 10 displayed are correct. FORTRAN programmers do the same thing when they display the value of a DOUBLE PRECISION variable (~15 decimal digits) out to 8-10 places. Because calculators are used only for numeric calculations, great care goes into the construction of the algorithms used for computations. Tests such computing SIN(x) followed by ARCSIN(x) [user tries different values of x] and then subtracting x off should result in a zero value.

Programmable calculators measure their memory size in terms of "number of memories" or instruction "steps". A "step" is a single instruction such as addition, multiplication, SIN(), SQRT(), subroutine call, etc. A "memory" location can hold one floating point number. For the HP models, 7 steps is the same as one memory; for the Casio models it is 8 steps = 1 memory. Memory size has increased greatly over the years. The earlier models had 128 to 512 steps of programming space. Today machines range from 4096 steps up to over 32768 steps. The programming language on a programmable calculator is like assembly language. Included are LABEL, GOSUB, ISZ, etc. instructions for program control. The main feature is that the codes are entered just by pressing the keys for performing the desired operations in the correct order. The more advanced calculators use letters A-Z for memory locations (like BASIC); earlier models used numeric locations.

The biggest advantage calculators enjoy over computers (outside of their small size and low cost) is the amount of built-in "smarts". Most high-end scientific programmable calculators have the ability to perform matrix operations, numerical integration, solve differential equations and plot rectangular & polar plots. All of this is stored in the machine's ROM (Read Only Memory). The user can call these functions from his program and thus write programs to solve complex problems with little effort.

Speed is the biggest drawback with calculators. There are many ways to measure computer speed. For our use here, the measurement is in FLoating-point Operations per Second (FLOPS). For modern computers the prefix Mega (10^6) is used. In literature involving supercomputers such as CRAYs and CONVEXes, the peak MFLOPS figures are given. This is jokingly known as "Macho-FLOPS". A well-known benchmark called LINPACK {07.02} is used to obtain MFLOPS ratings. LINPACK concerns itself with matrix calculations. There are no transcendental function (SIN(x), LOG(x), etc.) calculations used in LINPACK. Figure 07.01 shows the speed of different computing devices. Note that all values are approximate. For computers, 64-bit numbers are used.

Machine	MFLOPS	Notes
Casio Fx-7500G	0.0002	Fast graphics calculator
IBM-PC (8086/87)	0.012	Original IBM-PC
286-12 w/ 287	0.028	From 1984
VAX 11/780	0.14	Original VAX from 1977
386DX-25 w/ 387	0.25	First 32-bit home computer
PDP-10	0.33	DEC mainframe {07.03}
486DX-33	1.4	Popular CPU for PC
Pentium 90	7.7	Current (1995) PC
SGI R4400/200	16	UNIX based workstation
DEC Alpha AXP 7700	40	High end minicomputer
CRAY C-90	1000	Supercomputer with 1 CPU

Fig. 07.01

MICROCOMPUTERS

Calculators gave the general public a "taste" of personnel computing. In January 1975, the first kit home computer, the ALTAIR was featured on the cover of "Popular Electronics" [a popular magazine in the U.S.A.]. This machine was aimed at electronic hobbiests who wanted to build a simple 8-bit computer. The memory was only 256 bytes (yes... a quarter of a K) and one entered programs by use of toggle switches. Two years later the APPLE and PET computers could be purchased for ~$1000 (US). These machines had BASIC in ROM and 8K bytes of memory.

Until the IBM-PC was unvailed on 12 August 1981, small 8-bit home computers were viewed as "toys" (because they ran games) by the business world. Even the more advanced machines running CP/M (An operating system that existed before MS-DOS) were looked upon with suspicion as well. Early versions of BASIC on these machines supported only single precision calculations which meant only 6-7 decimal places of precision. The accuracy was poor as well as the writers of the BASICs encountered were more concerned with writing a general purpose language then in providing a serious scientific tool. The one exception to this was the HP-85 with a very powerful version of BASIC that supported graphics and 12 digit numbers with a dynamic range of 10^(+-500). It was very expensive.

The IBM-PC made small microprocessor based computers "legit" in that "big-business" purchased these machines in droves. IBM also published data on how the machines worked so third party hardware & software vendors can make products for this class of machine. With IBM's marketing muscle; CPM based machines disappeared overnight. IBM selected Microsoft to write their PC-DOS (what we today call MS-DOS) and selected Intel's 8088 microprocessor. The sequence of Intel processors, 8088-->286-->386-->486, etc. resulted in great increases in performance. All x86 microprocessors before the Intel 486 performed all floating point calculations in software which resulted in very poor FLOPS ratings. A special chip called a co-processor had to be installed to perform the calculations in hardware. They were named the same as the main processor except the number ended in a "7" instead of a "6" as in 8087-->287-->387. The focus of 8088, 286 & 386 machines was on office data processing such as word processing and spreadsheets. The quality of early PC software was very poor due to many software companies (and individuals) wanting to "get in on the act". By the time the Intel 486 appeared in 1989, the quality started to improve. The 486 was the first Intel microprocessor to have a built-in co-processor. It packed the computing "fire-power" of an IBM-370 mainframe. Mathematical/scientific programs such as MATRIXX, MATLAB (numerical matrix calculations), MATHCAD and MAPLE appeared. Mainframe quality compilers for FORTRAN, C and PASCAL from companies such as SVS, Lahey and Microway became within reach (costwise) for the interested public. Researchers are now able to obtain mainframe class (1970's mainframes that is) performance for little cost.

Scientists and mathematicians comprise less than 10% of the computing population. What has fuelled the big demand for PC power is not from the scientific community but from the graphics community. Computer games are graphics intensive. Games like DOOM {07.04} and Wolfenstein3D use complex 3-D graphics to give the player a feeling that he is in another place. The 2-D games like the 1982 hit PACMAN, are passe. There are many companies specializing in constructing "Video Accelerators" which are computer boards with special processors to handle drawing the screen display so the main CPU does not have to perform that function. It would not surprise the authors to see RISC-based vector processors {07.05} used in video boards. This would greatly increase the speed of the graphics.

Today, one can buy for under $2000.00 (US) a home computer that has more computing power than a CYBER-7600 (60-bit CDC large mainframe which was the fastest machine in 1969 before the CRAY-1 appeared). The latest Pentium processors run at over 300 MHz which delivers more than 25 MFLOPS of performance. This ability allows PC class computers to run multitasking operating systems such as Linux. Linux is the UNIX operating system for PC's. It's advantage are several: It is based on a system well known to scientists and engineers and it is (near) free. A software group called GNU have written free C, FORTRAN and (yes...) Ada compilers to run under Linux. A multitasking system allows more than one user to use the system at one time or a single user to do several things at once, just like on a traditional mainframe.

The real power of computers lie not only in their calculating speed but in their ability to manipulate symbols (information) very quickly. In the mid 1960's a group of mathematicians and computer scientists at MIT developed a program called MACSYMA. MACSYMA had the ability to solve mathematical problems by symbolic means instead of using numerical methods. The program was written in LISP which is a language used for list manipulation. MACSYMA "knows" basic mathematical rules like X*(Y+Z) = X*Y + X*Z. By using these, it can answer questions like,

```
/2
|  dx
| ----- = LN(3)-LN(2) = LN(3/2)
| (x+1)
/1
```

By employing basic integration rules, the program can return the answer in non-numeric form. The MIT researchers gave the initial version of MACSYMA the freshman calculus final exam to work on. MACSYMA scored over 90%(!).

Today, the best known symbolic algebra systems are Mathematica (Wolfram Research Inc), MAPLE (Waterloo Software) and MACSYMA. These systems compete in the market place and they are still at the stage of development where it is still possible to devise a set of problems that will cause all but your favorite system to fail. However, these programs are useful in that they free the mathematician from boring symbolic calculation much as the pocket calculator frees one from doing arithmetic. If it wasn't for programs like these, a computer is really just a giant programmable calculator whose memory is measured in millions of "steps", is >10000 times faster, and is programmed in FORTRAN, BASIC, etc.

The foregoing was an overview of the types of computing devices developed over the years and how they have improved in performance. Early Wexzal work was on mainframes. The modern PC has, if nothing else, brought mainframe power to the general public. Of course, mainframes have moved up in performance as well. The entire computing spectrum (from calculators to supercomputers) all have moved up the performance curve so much in the past 10-20 years that today's calculators occupy the performance level of low-end 1970's minicomputers and today's workstations are 1970's supercomputers.

An interesting thing to note is the difference in how a modern programmable calculator and a modern laptop computer fill the need for a mathematical computing device. Ignoring the difference in size and computing power for just a moment, a programmable calculator has most of its' abilities "built-in" in ROM. A laptop is more software oriented in that a laptop is not built "knowing" how to perform matrix operations; one needs to buy software in the form of a mathematical package like MAPLE or a programming language compiler such as FORTRAN. This ability to "install" a program or language of your choice gives the user far greater flexibility than a calculator where the user has to accept the "language" and/or interface presented by the calculator. In the case of a programming language, the user can "home-brew" his own routines to perform matrix calculations, least-square calculations, etc. It is now up to the user to insure correct operation of his programs. In the case of the calculator, the user just needs to be sure that he has entered the data correctly. Bill Gates, the CEO of Microsoft, has described this as the "trend to 'softness'". Programmable calculators are the last bastions of "hardness".

What does this have to do with Coupled Root calculations? Calculators and mainframes were used for the bulk of the author's early efforts. At first glance, this would appear strange until one observes that mainframes and calculators have the ability to calculate to high precision.

There are two other numbers along with dynamic range and speed that describe the performance of the machine. They are the number of significant digits (or bits) and "machine epsilon". The number of significant digits tells the maximum number of digits (in a floating point number) the machine uses during a calculation. On most machines, a DOUBLE PRECISION number has 15 significant digits. Calculators have (on average) 13 digits with some of the newest models such as the Casio fx-9700GE having 15 digits. The reason for such high precision is to guard against round-off errors. As a contrast, a slide rule is good for 3 significant digits which reflect real-world analog measuring resolution. Most analog scales such as meters (on electric equpment or cars) are good for 2 digits. The other number of importance is the machine epsilon. Computers compute using a finite number of digits so their resolution is finite. The machine epsilon is defined to be the largest number, EPS, such that 1+EPS=1. This is related to the number of significant digits used by the machine. EPS is given as a decimal number or as a power of 2. Most IEEE-754-1985 compilant computers have a machine epsilon of 2^(-53) = 1.110223E-16. If we compute the logarithm of this number (and ignore the sign) we obtain the number of significant digits. In this case it is log(EPS)=-15.9546 => 15 digits. A Casio fx-9700GE has a value of EPS=8.0E-14 which gives the number of digits = log(EPS)=-13.097 => 13 digits. For this calculator, this says the calculator can resolve to within 13 orders of magnitude in spite of the fact that 15 digits are used to perform arithmetic. Because Wexzals use logarithms, a test of the logarithm function needs to be made on the calculating device. The value of the logarithm should be correct to +/-1 digit in the least significant digit over the entire range of the logarithm function. This is one area where calculators were better than PC type machines until recent times. The early home computers such as the TRS-80, CBM PET-2001 and others had EPS=5.96E-8 => 7 digits. In the author's view, this was not going to "cut it" in spite of these machines having an easy-to-use BASIC language.

The performance and limitations of programmable calculators (slow speed and dynamic range of 10^(+-99)) influenced the approach as to efficiency (tight, elegent algorithms) and precision.

COUPLED ROOT CALCULATIONS

Coupled Exponents increase in size very quickly. What this means is that high precision is required to reduce the amount of error generated by rounding, etc. For example,

```
    CXT(7.0000) = 823543.0000                         (07.01)
    CXT(7.0001) = 823785.6447
    CXT(7.0010) = 825972.7197
    CXT(7.0100) = 848170.7786
```

This is due to this simple fact:

```
  d   x    x
  -- x  = x * [1 + ln(x)]                             (07.02)
  dx
```

To see this effect, a number called the "Condition Number" is used. The condition number is used to tell how much the output value of a function varies with respect to a given change in the input. That is,

```
  dy      dx             x   dy
  -- = C * --   where C = - * --                      (07.03)
   y       x             y   dx
```

Condition numbers are used in control theory where one wishes to analyze a system (circuit, etc.) to determine if it is "well-conditioned" or "ill-conditioned". An ill-conditioned system is one where a small relative change in the input (e.g. +/- 1%) causes a large change in the output. It would be nice if the change in output varied less than the change in input.

For the Coupled Exponent, C=x*[1+ln(x)] which says the "system" gets more ill-conditioned the larger the input is. This situation places great demand on precision (see discussion above) and accuracy of the logarithmic function. Some numerical analysis texts state the following as a rough rule-of-thumb concerning condition numbers: The number of digits lost due to rounding (on a finite precision machine) is approximated by log(C).

The first question is "How to compute a Coupled Root?".

There is no way to compute a coupled root in a fixed number of steps (without using Wexzals). This means sometype of iteration method is called for. We try,

$$y = x^{x} \qquad (07.04)$$

$$y^{1/x} = x \qquad (07.05)$$

We then start with x=1 (or better value if we know it) and then iterate using (07.05). The problem with this is that the larger y is, the more important it is to have a good initial value for x. The reader can test this for himself.

Another method is to use Euler's Sequence (see chapter 02) but with a modificaton: Euler's sequence is valid for 1/e^e<=1/y<=1/e^(1/e). This limits us to compute coupled roots for 1<=y<=15.+ which is not very useful. The one change is to compute a geometric mean between successive passes. So instead of (in pseudo-BASIC),

```
PRINT "EULER METHOD OF COUPLED ROOTS"
INPUT Y
Z=1/Y
A=Z^Z
FOR I=1 TO large_number
A=Z^A
NEXT I
ANS=1/A
PRINT ANS
END
```

We use,

```
INPUT Y
Z=1/Y
A=Z^Z
FOR I=1 TO large_number
B=Z^A
A=SQRT(A*B)
NEXT I
ANS=1/A
PRINT ANS
END
```

This broke the Euler "barrier" and provided a method of computing coupled roots. The main problem was now speed of convergence. This idea of using a geometric mean was found to be useful so a notation was invented to aid in manipulating this.

THE "XI"-OPERATOR

The Xi operator is a notation used for describing the forgoing algorithm. Because of the SQRT() step, the Xi notation can only be used for equations that have a positive root. The reason for "Xi" is that it is the Greek letter for X and is used much like Sigma (summation) and

Capital Pi (for products). The general form is,

$$y=f(x), \tag{07.06}$$

$$x = \Xi_{z=a}^{inf} g(z) \tag{07.07}$$

The "z=a" means to start with an initial value of a. The number on top of the Xi tells the number of times to iterate. The g(z) is a function of both x and y. An example would make this clear. Using (07.05) we have,

$$y = x^{x} \tag{07.08}$$

$$y^{1/x} = x \tag{07.09}$$

$$x = \Xi_{z=1}^{inf} y\hat{}(1/z) = \mathrm{crt}(y) \tag{07.10}$$

Another example would be the solution of x=cos(x). This is already written in iterative form and has a solution in 0<x<1.

$$x = \Xi_{z=0}^{inf} \cos(z) = 0.7390851332... \tag{07.11}$$

When one wants to write an equation in iterative form, it is best to use the most "powerful" inverse as this will aid in convergence. Solve,

$$\mathrm{cxt}(e\hat{}x) = 1 + \cos(x) \tag{07.12}$$

For x=0, cxt(e^x)=1, 1+cos(x)=2.   This means LHS < RHS.
For x=1, cxt(e^x)=15.15..., 1+cos(x)=1.54   This means LHS > RHS.
So a solution exists 0<x<1. We can write (07.12) as either,

$$x=\arccos[\mathrm{cxt}(e\hat{}x)-1] \tag{07.13}$$

---or---

$$x=\ln\{\mathrm{crt}[1+\cos(x)]\} \tag{07.14}$$

We chose (07.14) because of the combined effect of the natural logarithm and coupled root. Our solution is,

$$x = \Xi_{z=0}^{inf} \ln\{\mathrm{crt}[1+\cos(z)]\} = 0.4239850757... \tag{07.15}$$

The Xi operator has, at best, linear convergence. It is, however, very robust and compact. For 1979-1981 era programmable calculators with their 128-step to 512-step memories, this space efficiency is important.

WEXZAL CALCULATIONS

Computing coupled roots via Euler's method (for the limited range) or the Xi operator on a programmable calculator pose one problem: The biggest number that we can obtain the coupled root of is 10^100 which is just under 57.

$$crt(10^{100}) = 56.96124843... \qquad (07.16)$$

This range was too limiting for use in investigating the quasi-logarithmic behavior of coupled roots (See chapter 06). Wexzals (See chapter 03) were defined to "walk-around" this limitation. Since Wexzals are really just coupled roots of big numbers, a method of calculating them that was robust (did not care too much about starting values) and compact was needed. Using the Xi operator we have,

$$wzl(x) = \Xi_{z=2}^{\infty} \frac{x}{\log(z)} \quad \text{for } x>2, \qquad (07.17)$$

Why z=2? We want to get as close to zero as possible but still be able to compute Wexzals of large numbers.

This simple act enables one to compute coupled roots of numbers too large to store in a programmable calculator. (07.16) reduces to,

$$crt(10^{100}) = wzl(100) = 56.96124843... \qquad (07.18)$$

One can now go up to wzl(10^100) = 1.020317217E+98

SPEED-UP OF WEXZAL CALCULATIONS

The Xi operator is robust but slow in convergence. This slow speed is fine if time is not an issue or if only a few values are needed. To be able to compute many Wexzals for calculations, such as for numerical integration, higher efficiency was required.

One of the best known numerical methods for solving equations is the "Newton-Raphson" method. That is,

$$f(x) - y = 0 \qquad (07.19)$$

$$x = x - (f(x)-y)/s \quad \text{where } s = \frac{df}{dx} \qquad (07.20)$$

Like the Xi operator, an initial value of x must be chosen. Here is where the problem lies. The Newton-Raphson method is quadratic convergent which means for each iteration the number of correct digits doubles. This is "wunderbar" except the method is very sensitive to the initial value of x. If the initial value for x is too far from the root of the equation, the method will diverge. So the question is: How do we automate the selection of the initial value for x when computing Wexzals? The solution for this lies in using an approximation method devised to enable one to compute Wexzals on small non-programmable calculators.

The Wexzal was defined in February of 1981. The asymptotic property (See chapter 03) was discovered and proved later that month. The following month, a method was devised for approximating Wexzals over a small interval by use of a 4-banger calculator that could compute square roots. Such machines were very cheap ($10) and in common use. This method was also very fast when high accuracy was not needed. This is how the method was derived: When one plots the Wexzal function over the interval [0,10], it looks something like SQRT(x) in that its rate of increase slows down. Maybe the Wexzal can be represented by a sum of square roots or something to that effect. We try,

```
    wzl(x) = a*x + b*sqrt(x) + c*x^(1/4) + d*x^(1/8)
                                        for x in [p,q]         (07.21)
```

There is nothing special about using 4 terms, the initial idea was tested with the first two terms. We then select the interval to be [1,number_of_terms] or in this case [1,4]. To compute the coefficients, [a,b,c,d] we solve a 4x4 system of equations. A PDP-10 computer with BASIC was used for calculating the solution of the following system:

```
    | 1 sqrt(1) 1^(1/4) 1^(1/8) |   | a |   | wzl(1) |
    | 2 sqrt(2) 2^(1/4) 2^(1/8) | * | b | = | wzl(2) |         (07.22)
    | 3 sqrt(3) 3^(1/4) 3^(1/8) |   | c |   | wzl(3) |
    | 4 sqrt(4) 4^(1/4) 4^(1/8) |   | d |   | wzl(4) |
```

For this we get,

```
    a =+0.347487883
    b =+3.180799039                                             (07.23)
    c =-4.877526987
    d =+3.855424212
```

Today, this calculation can be made on any calculator with built-in matrix calculations such as a Casio fx-7700G.

Letting g(x) = a*x + b*sqrt(x) + c*x^(1/4) + d*x^(1/8) we compute the Root Mean Square (RMS) over the interval [1,4],

```
          /4
          |
    RMS = | sqrt{[wzl(x)-g(x)]^2/3} dx = 1.51184E-05           (07.24)
          |
          /1
```

Which is not bad for a simple expression that allows one to compute coupled roots in [10,10^4].

Experimentation with (07.21) shows that this approximation does not "fall-apart" until x is around 15. For x>15 we can use the (crude) approximation wzl(x)=x/log(x). For x in [0,1) we use the Taylor series,

```
        wzl(x)=1+x/m+...                                        (07.25)
```

So a complete algorithm would be the following:

```
        REM To compute wzl(x) for a given x
        m=LOG(e^1)
        IF 0<=x<1 THEN LET y = 1 + x/m
        IF 1<=x<=15 THEN LET y = a*x + b*SQRT(x) + c*x^(1/4) + d*x^(1/8)
        IF x>15 THEN LET y = x/LOG(x)
        FOR I=1 TO 16
        z=LOG(y)
        y=y-(y*z-x)/(m+z)
        NEXT I
        RETURN y
        END
```

A minor variation of this is currently used for calculating the values of the Wexzal function. This algorithm is fast, compact and simple to understand and implement.

WEXZALS OF COMPLEX NUMBERS

Wexzals of complex numbers (and reals that are < 0) can be computed in the same manner except one needs to use ABS(x) in place of x. With this, one will obtain values that are in the complex plane. Examples:

```
    wzl(-1) = crt(0.1)  = -0.2063655338 - 1.299526203*i         (07.26)
    wzl(-2) = crt(0.01) = -0.8152336827 - 2.031576410*i
```

The equation,

```
    x = log(x)                                                  (07.27)
```

has the solution,

```
          1
    x = ------- = -0.1191930734 + 0.7505832939*i                (07.28)
        wzl(-1)
```

A GEOMETRIC METHOD

Wexzals of positive numbers can be obtained via graphical means by plotting on a linear graph the two equations y=log(x) and y=k/x where k>0. The point where they intersect is (wzl(k),k/wzl(k)). This is simple as the logarithmic graph is "standardized" and only the value of k need be varied.

WEXZALS ON A SLIDE RULE

Slide rules have become "hot" collector's items in recent years. In spite of the digital "juggernaut", slide rules have a charm of their own that refuses to die. This we think is due to the user being able to develop a "feel" for the numbers and their relationships as expressed in the different scales. Some experts can mentally perform calculations in their heads by imagining a slide rule in operation.

A slide rule looks like a fancy ruler and has three parts: The body which is the main part; a slide which is the center bar that moves within the body and the cursor which is the clear indicator with a vertical hairline. There were many different types made such as circular models and Deci-Trig models by Keuffel & Esser (K&E). The most common type is the Mannheim {07.06} which has all 9 scales on one side. The other side has conversion factors and other mathematical/physics reference information. We do not intend this to be an introduction into slide rule operation. For that, a public library that still stocks "old" books will have books on slide rules. They can be located in the mathematics/physics section. A retired engineer or scientist who has ended his career before 1975 {07.07} can be of great assistance.

Wexzals can be calculated on a slide rule provided the L scale and CI scale are on the body and slide and the L scale runs from left to right. Some L scales run from 1 to 0 instead of the (more common) 0 to 1. The basic idea is to find where CI and L have the same value for a given slide setting. This common value will be log[wzl(x)]. As an example, to compute wzl(1) we have the slide rule "closed" (The "1" on the C and D scales are over each other) and move the cursor from left to right and at the same time looking to see where the value on the CI scale and L scale will be the same. It is at 0.399 If the cursor is to the left then the value on the L scale will be less than the CI scale. If the cursor is to the right of 0.399, the value of L will be greater then CI. Look at the value on the D scale for the final answer. It will be about 2.51, this is wzl(1). To compute wzl(2), we move the left hand "1" of the C scale over 2.0 on D and repeat the same process. The answer is wzl(2)~3.6. This is fine for x in [1,10] but how do we compute higher wexzals? The CI scale can represent any number; not just numbers in [0,1] like the L scale. We multiply CI by the correct power of 10 and by using the fractional part of CI we make the same test against the L scale. For example, to compute wzl(20), we know it is around 15 or 16 (use the crude approximation wzl(x)~x/log(x)) so we move the right 1 of C over the 2.0 on D and now CI represents numbers running from 1 to 10 as log[wzl(20)] > 1. Using the fractional part, L and CI meet at 0.216 on L. The CI reads 1.216 but the 1 tells us to multiply the answer by 10^1. The D scale says 1.64 which when multipled by 10^1 gives 16.4 which is wzl(20). As a final example, compute wzl(4000). Well... 4000/log(4000) = 4000/3.6 ~ 1100 => log[wzl(4000)] is over 3.0 so we know that the power of 10 to multiply by will be 3. Move the right 1 of C over the 4 on D and scan between the 3.0 and 4.0 on CI. The CI and L meet at 0.11 on L. Read 1.28 on D. The final answer is 1290. The actual full answer is wzl(4000) = 1286.428 By playing with this, the reader can determine how to compute wexzals of very small numbers. Hint: Use the asymptotic expression wzl(1/x)~1+1/(m*x).

CONCLUSION

This chapter showed the development of the methods used to compute numeric values for the coupled root and Wexzal function. A brief overview of the history (and authors experiences with) of calculators and early home computers was also presented. Numeric calculation was central to early Wexzal research. This is why much effort was directed towards this end. By 1983, a least-squares method involving Wexzals was developed. This involved hundreds of Wexzals being computed per problem so efficiency was of upmost importance.

07.01:

"Reverse Polish Notation" (RPN) is not a joke; it is a modification of a notation invented by an 18th century Polish mathematician. His notation uses prefix operators whereas RPN uses postfix notation. A brief example would be,

```
X + Y * Z = X Y Z * +
```

where the numbers are "pushed" onto a stack and then popped as each function is performed.

Hewlett-Packard uses this system on their calculators. It has the advantages of giving the user great control over the state of the calculator. Short-cuts can sometimes be effected by "rolling" or swapping numbers on the stack. All of the pre-HP-48 machines have a 4-level stack. Note that these machines do not have an "=" key. HP calculators enjoy a very high reputation due their workmanship and excellent support from Hewlett-Packard. Critics and many beginners find the machines over-priced and difficult to use.

Casio, Sharp and TI use what is called "Algebraic Operating System" (AOS). This is the familiar system where one enters a calculation the exact same way as one would write it. So our example would just be

```
X + Y * Z =
```

and upon pressing the "=" key, the answer would appear. AOS calculators really convert their internal calculations into RPN before performing the calculation. This is how the calculator "knows" to perform multiplication before addition. This is why there is a limit to the number of embedded expressions (formulas in ()) allowed. Try hitting the "(" too many times and the machine will lock-up or give an error indication. AOS is easier for beginners to master. Casio calculators enjoy a following also due to their ease of use, speed, innovation (they were first with a graphics model) and low cost. Calculators from the "big-4" (Sharp, TI, Casio and HP) are all very reliable and give the user cost-effective computing ability.

07.02:

LINPACK is a FORTRAN linear algebra benchmark written at Oakridge National Labs. It is used to measure the arithmetic (add, subtract, multiply and divide) speed of computers. Matrix calculations are required to solve dynamic problems in nuclear physics. This benchmark has been used to measure the speed of machines from PC's to CRAY supercomputers. There are two problem-set sizes: 100x100 and 1000x1000 matrices of 64-bit (DOUBLE PRECISION) numbers. The larger problem is used to measure the efficiency of vectors on supercomputers such as the Cray YMP-C90.

Benchmarking is a tricky subject in that there are many different ways to benchmark a machine and tests can be arranged as to make the machine of your choice win. Industry standard benchmarks such as SPEC, AIM, LINPACK, etc are attempts by major computer vendors to give objective "horsepower ratings" of their machines. The best (and standard) answer to the question "which machine is best?" is to run a sample program that represents the type of work the machine is intended for. Use the results of that to make an evaluation.

With the rise of RISC processors such as the DEC 21164, MIPS-8000, etc come the fact that for these processors, the quality of the compiler used to generate machine code is very important. The compiler must generate efficient code that takes full advantage of the features of the processor like multi-piplines, caching, etc. Both the processor and compiler need to be viewed as an integrated system.

07.03:

The PDP-10 is one of the most beloved and famous of the DEC mainframe line. It appeared in 1969, ran the TOPS-10 (Total OPerating System 10) operating system and had a 36-bit word. It was one of the first machines to use timesharing on a wide scale. Schools, universities and government labs had these easy to use systems. PDP-10s supported BASIC, MACRO (DEC assembler) FORTRAN and COBOL. Being a "word" machine, it was most efficient running MACRO and FORTRAN.

TOPS-10 had the "feel" of OpenVMS that currently runs on VAXes and Alpha AXP systems. The designers of MS-DOS used many of the features of TOPS-10. Users logged onto a PDP-10 much the same way that one does on a VAX except a Project Programmer Number (PPN) is entered instead of a username. The PPN is a pair of three digit octal numbers that is used to identify the "group" and user id of each user. This is where the VAX UIC (User ID Code) came from. Each user ran in timesharing mode and was allocated (at most) 1 MB of core (yes... the machine had actual magnetic core elements) for his process. All of the major subsystems had processors on them to free the CPU to perform the "important" work. The PDP-10 was front-ended by a PDP-11 which communicated with the terminals. All of this made for efficient timesharing. TOPS-10 was a very solid operating system that had one of the most advanced scheduling algorithms in use.

In terms of raw arithmetic speed in DOUBLE PRECISION (72-bits), a PDP-10 (with the KL-10 processor - the fastest and last of the series) rates about 0.4 MFLOPS which is around the same performance of a modern Intel 386DX-33 based PC. By 1970 standards, this made the PDP-10 a "small" mainframe. An average PDP-10 system cost $500,000 (1970's dollars). The "big-boys" were the IBM-370/168 and CDC-6600 machines. These big machines were more for batch processing in large business (IBM) or scientific (CDC) sites. The PDP-10's main strength lies in its ease of use. High school students could use the PDP-10 with little problem. IBM's JCL language, as used on the IBM-360 and IBM-370 series, was very difficult to master. IBM tried to make their own timesharing operating system called TSO which never achieved the popularity of TOPS-10.

DEC's last entry in the mainframe world was the DECsystem 20. This machine was really a PDP-10 with a new operating system called TOPS-20 and instead of core memory it had solid-state memory. TOPS-20 is a major upgrade on TOPS-10. It had sub-directories, usernames (no more PPN's) and a feature that when the user would type part of a command (or filename) and then strike the ESC key, the system would (if it could) finish typing the rest of the command or filename. This made for a system that could be best described, from a user's point of view, as "silky-smooth". The largest machine of this series was the DECsystem 2080 which had over 2 million words (36-bit) of memory. The VAX-11/780 in late 1977 lead to the demise of these mainframes as DEC wanted to be a one computer "family" (VAX) company. The VAX used the best ideas from the PDP-11 (DEC's 16-bit mini) and the PDP-10. Today (1995) the VAX is on it's last legs as DEC has moved on with the Alpha AXP family. This 64-bit system family is one of the fastest non-supercomputer systems to be had today.

Both authors learned computing in high school on a PDP-10 in the mid 1970's and enjoyed the experience. Early coupled root tables were generated on a PDP-10 in BASIC and FORTRAN.

07.04:

DOOM and WOLFENSTEIN3D are two very popular computer games that have earned awards due to innovative use of 3-D computer graphics. Unlike the 2-D format of PACMAN, Wolfenstein 3-D places the player in a dark German castle in Germany during WWII. The view is exactly like what one would see in real life (perspective). In the view, the player sees a hand holding a weapon such as a dagger, P-38 pistol, a 9-mm Schmeisser machine-pistol or an 8-mm machine gun. On computers with multi-media (soundcard) the sound effects are so real, the player really feels like he is in the middle of WWII. Music is used to add to the "atmosphere". It is clear that the creators at ID Software did their homework in researching this game as the opening march is "Horst Wessellied" which was the offical Nazi march of that era.

The player tries to move his character from floor to floor to escape German solders, SS guards, etc. When encountered they yell "Achtung! Halt! Schuezstaffel!" before they start shooting. All of this action in the game require great computing and graphic power. Many of the scenes used are pre-computed but when the character moves down a hallway it is clear that computational power is needed to render the scene based on character movement. An interesting observation is that all dead solders, guards, etc lie on the ground with their feet pointing to the player irrespective of the player's location. One can shoot a solder, see him fall so his feet are seen. The player can walk past him then turn around to look back on him and see that the feet are still pointing towards him.

DOOM is like Wolfenstein3D except it takes place on Mars sometime in the early 21 century and the player fights demons and monsters instead. The action is much more intense and the player can jump stairs, walls, etc and shoot at different elevations. DOOM is really a major upgrade on Wolfenstein3D.

Both these games make great demands on the graphic subsystems on PCs. Graphics cards (sometimes called Graphics Accelerators) are cards that contain a processor to handle graphics displays. This frees the CPU to perform the "important" calculations and let the graphics card handle the details of graphics. This, in essence, turns a PC into a baby-mainframe in that subsystems are "intellegent" enough to handle the details of their own operation with little intervention by the CPU. Realtime Multi-media is the driving force behind the demand for more powerful CPU's and graphic subsystems. The real goal is true Virtual Reality where one can simulate (in true 3-D) any situation they wish. It would not surprise the authors to see vector-processor graphics cards. A VGA display is a grid of pixels 640 x 480 which if represented by a 640 x 480 matrix would be ideal for a Cray-like vector processor. Such a system could perform in the hundreds of MFLOPS and be used for true realtime graphics rendering.

07.05:

Vector registers enable machines like the Cray YMP-C90 and NEC SX-3 to perform calculations on an array of numbers in the same manner as scalar calculation (one number at a time). Thus a cross product calculation (used in matrix multiplication) can be done in just slightly more time (allowing for overhead) than a scalar multiplication. It is this vector ability that give Crays and other supercomputers their impressive speed ratings. When forced to calculate in scalar mode, their speeds are not that much better than a high-end workstation. Most computing experts would agree that for cost effective scalar computing, it is better to use a machine like a DEC ALPHA 7700 AXP or SGI PowerOnyx workstation.

07.06:

Andre Mannheim was an officer in the French Army during the early 1800s under Napoleon. Napoleon liked mathematicans and mathematics and though mathematics useful. Mannheim was assigned the task of standardizing all aspects of French artillery. Artillerymen need to make quick calculations in the field (correct firing angles, etc.). Mannheim standardized the slide rule for use in the French Army. This is one of the first cases of a "Mil-spec" being given for a non-weapon item within an army. Mannheim's slide rule has 9 scales (S,K,A,B,CI,C,D,L,T) on one side and reference information on the other side. This gives the user the ability to multiply and divide, compute square and cube roots, Sines and Tangents, Reciprocals and logarithms.

Today, the American Army has standards for laptop computers and computer languages. The best known of this is Ada-83 which is MIL-STD-1815A. The goal in both cases is the same: to enable the army to use a standardized item so a large number of men can use it and logistics (support) is simplified as much as possible.

07.07:

To show how quickly slide rules have been displaced by calculators the following is a true story. In 1981, one of the authors, while attending the university, left a plastic slide rule in the Computer Center early in the morning by mistake. After a day of classes, he was surprised to find it exactly where he left it. This was in an era where calculators (even 4-bangers) "grew legs and walked".

Chapter 08

Misc Items Involving Wexzals

INTRODUCTION

The following are topics that wrap-up "loose ends" involving the Wexzal function. They range from interesting open questions down to "trivia".

CONVERGENCE PROPERTY

In Spring 1981 it was discovered that we define the iterated Wexzal as,

$$wzl^{0}(x) = x \qquad (08.01)$$

$$wzl^{1}(x) = wzl(x) \qquad (08.02)$$

$$wzl^{2}(x) = wzl[wzl(x)] \qquad (08.03)$$

$$wzl^{3}(x) = wzl\{wzl[wzl(x)]\} \qquad (08.04)$$

We get the following result,

$$\lim_{n \to \infty} wzl^{n}(x) = 10 \quad \text{for all x in complex plane} \qquad (08.05)$$

This says that for any value of x in the complex plane, the iterated Wexzal values converge to 10. Not much has been done with this fact. An interesting graph is the plot of,

$$f(x) = \sum_{n=1}^{\infty} \{[wzl^{n}(x) - 10]^{n}\} \quad \text{for } 0 \le x \le 10 \qquad (08.06)$$

Some values are:

x	f(x)
0.000000000	-.4575750228
1.000000000	.7906822176
2.000000000	.2223556430
3.000000000	.0171801656
4.000000000	.0415736557
5.000000000	.1260693227
6.000000000	.1871547521
7.000000000	.2219561372

8.000000000	.2907658270
9.000000000	.5018692795
10.00000000	1.000000000

It was hoped that some kind of a series could be developed from (08.06) but it was not met with much success.

Chapter 09

Curve-fitting with the Wexzal

INTRODUCTION

Most early research efforts into the properties of the Wexzal function were focused on the basic theory of the Wexzal and Coupled Root functions. This included integrals (chapter 05), solving equations in closed form (chapter 04), etc. Nevertheless, there was the nagging question of "can this stuff be applied? If so, how?". The Wexzal function is interesting in of itself but there was yet the need to make it "useful". In April 1983 an interesting question arose which fuelled the need to develop a fast efficient non-linear curve-fitting algorithm that involved the Wexzal function.

MODELING

Applied mathematicians "earn their keep" by working as mathematical modelers for industry, government, etc. One hears and reads about the latest model for how the universe was created or how an economic system works. Exactly what is mathematical modeling?

Mathematical modeling is the act of using mathematics (formulae) to describe some activity in the physical world. By use of formulae, mathematicians hope to not only describe the activity under investigation but to make predictions about that activity via the formulae. The most difficult part of modeling is outlining the assumptions and limitations of the model. If these are understood, then one can use the model with some measure of confidence. Models are best used in situations where one either: cannot directly observe the activity (e.g. creation of universe) or would be a danger to get involved directly with the activity until it is better understood. The best examples of models that are examined before the dangerous activity is undertaken is in the field of drug research and flight simulation.

When a drug company develops a new drug, they model it on a computer. The basic behaviors of the human body are programmed into the computer and the properties of the new drug are then entered. Based on mathematical laws about the human body and the drug, the computer can predict the body's reaction to the drug. Assumptions and limitations on all of the models are carefully checked. Once it appears the drug will work, only then will the drug company test it on lab animals before testing it on humans.

An example is in the field of aircraft design. Flight simulation models are used to help design new airplanes. These same models are used in flight simulators used to train pilots. Here the goal is to accurately mimic an actual airplane. Dispite the impressive sights, sounds and reactions of modern flight simulators, many pilots place little faith in these devices because the limitations and assumptions of the simulator are not always spelled out. There are many stories of Air Force pilots training on simulators (to learn a dangerous maneuver) only to find that the actual airplane, when flown over 50,000 feet, does not fly like the simulator. This difference could be caused by incorrect/missing numerical data for that flight altitude. In spite of that, simulators and other modeling devices save money, time and (most important) lives.

The models we are concerned with here are much simpler than a flight simulator. We are attempting to describe an event or action with a formula that has one or more parameters. These parameters are used to control the behavior of the basic formula.

The first part of modeling is to decide on the form of the formula(e) to be used. This is based on either known physical laws (e.g. Gravity law) or it could be a "best guess" by the modeler. In this case, the modeler uses the formula to make predictions about the activity under investigation. Experiments are performed (if possible) to validate the model and to find the conditions where the model is not valid.

In most physics textbooks there is a discussion on falling objects. The acceleration of all falling objects is constant. For the model,

$$v = k * t, \quad \text{where } v=\text{velocity},\ k=\text{acceleration},\ t=\text{time} \qquad (09.01)$$

$$x = 1/2 * k * t^2, \quad \text{where } x=\text{distance} \qquad (09.02)$$

one finds that this model is true if air resistance is removed. For experiments in air, this model would not be valid as a cannon ball will fall faster than a feather due to air drag. In a vacuum, this model would be valid. Once the limitations have been noted, these formulae can be manipulated to give "new" facts about falling objects. A very simple example is computing,

$$\frac{dv}{dt} = k \qquad (09.03)$$

This says that the acceleration is constant. Experiments can be performed to aid in calculating this value.

CURVE-FITTING

Many times, a modeler has the basic formulae for the model but lacks needed parameters (such as 'k' in (09.01)). This is in contrast to most of applied mathematics where the formulae themselves are unknown. The best known way to obtain the value of the needed parameters when given the formulae and a set of observations is to perform a calculation known as "Curve-Fitting".

Curve-Fitting is the act of finding values for the parameters in a formulae such that,

$$(f[x(i),a,b,c,...] - y(i))^2 \quad \text{for } i=1,2,...n \qquad (09.04)$$

is as small as possible. Note that a square quantity is used. If this difference is zero for i=1,2,...n then the formula matches the readings [x(i),y(i)] exactly. Most of the time, this does not occur.

Many equations can be reduced to linear form. A linear equation is in the form of,

$$y(i) = A * x(i) + B \quad \text{where A,B are unknown constants} \qquad (09.05)$$

Most curve-fitting problems involve equations that can be placed in this form by simple transformation of the (x,y) data. The four main forms are:

$$y = A*e^{B*x} \qquad \text{[Exponential - use (x,ln(y))]} \qquad (09.06)$$

$$y = A*x^{B} \qquad \text{[Power - use (log(x),log(y)]} \qquad (09.07)$$

$$y = A*\log(x) + B \qquad \text{[Logarithmic - use (log(x),y)]} \qquad (09.08)$$

$$y = A*x + B \qquad \text{[Linear - use (x,y)]} \qquad (09.09)$$

Scientific calculators (see chapter 07) have routines built-in to perform linear curve-fitting. It is sometimes known as "Linear Regression".

NON-LINEAR FORM

Equations that cannot be placed in the forms of (09.06) thru (09.09) are said to be of non-linear form. An example of this is,

$$y = \frac{A}{wzl\left(\frac{B}{x}\right)} \qquad (09.10)$$

This equation arose from a study on the relationship between number of pumps on a pneumatic (pump-up) airgun and the muzzle velocity. Most airguns employing this power system are made by companies such as Daisy [09.01], Sheridan and Crosman. Most of these guns shoot 0.177 BBs (steel balls)

Chapter 10

Graphs for Wexzal Calculations

INTRODUCTION

Graphs for used to display functional relationships between the independant variable(s) and the dependent variable. Along with the standard x-y linear graphs there are others such as Polar graphs. In most engineering/technology work, logarithmic plots are used.

LOGARITHMIC GRAPHS

Logarithmic graphs are used in scientific/technical research for plotting experimental results or equations that involve logarithms. Logarithmic graphs are noted for having the sub-divisions on one or the other or both axes be logarithmic. A slide rule is divided the same way. The advantage of this is that data over a wide range can be plotted as each "count" along the logarithmic axis represents a 10-fold increase in magnitude. Each count (power of 10) is called a "decade". Thus data such as distances ranging from atomic distances (order of 1E-10 ft) upto intergalactic (order of 1E+24 ft) distances can be plotted on the same graph. Each decade is of equal size so it is just as easy to read the data for (in our example) distances in the solar system (1E+11 upto 1E+13 ft) as it is to read distances between cities (1E+3 thru 1E+8 ft).

If one were to try to make such a plot using linear axis, he would find that the upper part of the scale would dominate as the axis would run from 0 to 1E+24. If the axis were sub-divided into 1000 parts (this would make for a very fine-lined graph that would be hard on the eyes), each sub-division would represent 1E+21. Clearly the graph would be useless except for intersteller distances. Logarithmic graphs are useful but they also have some drawbacks.

If one knows the range of data to be plotted then the decade count can be made. However there is no "zero" on a logarithmic graph (along the logarithmic axis) as log(0) = -inf. One can select a very small number and let it go at that but what if the number selected is too large? E.g. In electronic control systems, engineers analyse systems based on frequency input. If the lowest frequency is lowered, then the graph must be re-plotted with the added decade. On the other end of the scale, the upper limit is bounded. One cannot plot "inf" on a logarithmic axis. There are ways to make axes that are non-linear and non-logarithmic that solve one or both these problems. This chapter presents two new types of graphs that solve both these problems and yet give new insight into the properties of the equation or data being plotted.

THE "FISH-EYE" GRAPH

The Fish-eye graph (a.k.a. "Sklar" {10.01} graph) is a graph that uses (on x-axis or y-axis or both),

```
       1
    -------- = <location of x on axis>                          (10.01)
    wzl(1/x)
```

to generate the axis. Assume that the graph to be made is square that is 1 ft by 1 ft. This description will be for the x-axis as the y-axis would work the same way except it is vertical. Call the left-most x-coordinate "0 ft" as it is 0% of the travel to the end of the axis. Call the right-most x-coordinate "1 ft" as it is 100% of the travel on that coordinate. Now to label the axis we compute (10.01) for each point we want to label. We make a chart:

Number label	Where to place on axis (ft)
0	0.0
1	0.399
2	0.538
3	0.621
4	0.677
5	0.718
6	0.750
7	0.755
8	0.795
9	0.812
10	0.827
15	0.874
20	0.901
30	0.931
50	0.957
100	0.978
1000	0.998
inf	1.000

(Fig. 10.01)

When one starts to contruct a such a graph one sees how sparse the axis is for 1,2,3, as "1" is located at about 40% of axis; "2" is about 54% and "10" is about 83%. The labels start to bunch-up as "100" is about 98% and "1000" is almost at the end. The appearence is that of a "distorted" logarithmic axis that somehow has "0" on it. The important thing to note is: Both 0 and infinity can be plotted at the sametime. This type of axis is called "Sklaric". There are many different ways to achieve the same goal of having 0 and infinity on the same axis, so why one based on (10.01)?

FIRST USE OF SKLAR GRAPHS

When initial research was done on the question "what is the relationship between barrel length and muzzle velocity?" (see chapter 12) the first thing done was to plot on a semilogarithmic graph the barrel length in inches vs. muzzle velocity in ft/sec. The goal was to see if the plot would result in a straight line which would indicate that the relationship was logarithmic. A standard research technique is to take the data and plot it on four types of graphs: (1) x-y axis linear, (2) x-axis logarithmic y-axis linear, (3) x-axis linear, y-axis logarithmic, (4) x-y axis both logarithmic. One of these should result in a straight line. From that, the researcher can tell if the relationship is linear, logarithmic, exponental or power. Modern scientific calculators such as the Casio fx-7700G, Ti-85 and HP-48SX have this feature built-in so all the researcher need do is to have the calculator perform the four types of curve-fits (which are just logarithmic transforms of the linear case) and check for the best correlation coefficient.

When the following data, for a .44 Magnum [10.01], was plotted, it was observed that it was "almost" logarithmic.

Barrel lgn	Muzzle vel
2.0	935.0
3.0	1067.0
4.0	1165.0
5.0	1239.0
6.0	1298.0
8.0	1384.0
10.0	1445.0
12.0	1490.0
14.0	1525.0
16.0	1552.0
18.0	1575.0

(Fig. 10.02)

What is meant by "almost" logarithmic is the plot is a straight line except for the last few points which start to bend downward. This indicated that eventhough logarithms would give a reasonable approximation to the data, it was felt that a different type of function would give a better approximation. The rationale for using (10.01) for this problem is given in chapter 12.

If one draws a graph with the x-axis being sklaric and the y-axis being linear ranging from 0 to 1 and then draws a straight line from (0,0) to (inf,1), one has plotted y=1/wzl(1/x). One can change the upper limit on the y-axis from 1 to a and then plot a straight line from (0,0) to (inf,a). This would be the equation y=a/wzl(1/x).

When one plots the data from Fig 10.02 on a semi-sklaric graph (x-axis is sklaric; y-axis is linear) one observes the data is very close to being a straight line. A Wexzalic curve-fit of Fig. 10.02 in the form of

$$y = \frac{a}{wzl(b/x)} \qquad \text{where a,b are constants} \qquad (10.02)$$

gives,

$$y = \frac{1779.692407}{wzl(1.1080842/x)} \qquad RMS = 5.77626 \qquad (10.03)$$

The b coefficient being "near" 1 explains the "straightness" of the curve.

One can obtain an approximate value for 'b' by observing the shape of the curve (of a sklaric function) on a semi-sklaric axis. By drawing a line from (0,0) to (inf,a) and noting the behavior of the function in relation to the line. If the function arcs over the line that means the function is "accelerating" to the asymptotic value faster than a/wzl(1/x). This implies that:

Function over line ==> 0<b<1 (10.04)

The closer 'b' is to zero (from the right) the more of a step-function the function becomes. This is because,

$$a/wzl(0/x) = a/wzl(0) = a/1 = a \qquad (10.05)$$

If the function curves under the line that means the function is slower than a/wzl(1/x) in going to the asymptotic value, 'a'.

Function under line ==> b>1 (10.06)

Examples of both cases include bullet acceleration inside of a gun (chapter 12) and automobile acceleration (chapter 13).

OTHER USES OF THE SKLAR GRAPH

Unlike logarithmic graphs that retain their "shape" regardless of the units used in the data (feet, miles, etc), Sklar graphs are not invariant in this respect. This at first looks like a draw-back but it can be used to advantage. By altering the units in the data one can make a graph that is easy to read and be useful in making complex calculations. An example of this is a Sklar graph for the equations used in solving the "car problem" (chapter 13).

A BETTER GRAPH FOR ASYMPTOTIC PLOTTING

One can plot (on the same graph) both zero and infinity. This is useful for asymptotic studies but there is just one problem: It is difficult to read values above 1000. There is not much difference (distance wise) between 1000 and infinity. We then cannot see distinct values for x=10000,100000,1000000, etc. There are many ways to solve this. If we give-up the ability to plot in the interval [0,1) then one way is to use,

$$1 - \frac{1}{crt(x)} = \text{<location of x on axis>} \qquad (10.07)$$

as the "generating" function. In chapter 02 it was pointed out that the coupled root function was "slower growing" than logarithms. We take advantage of this fact.

Number label	Where to place on axis (ft)
1	0.000
2	0.359
3	0.452
4	0.500
5	0.530
6	0.552
7	0.568
8	0.581
9	0.592
10	0.601
100	0.722
1000	0.780
10000	0.816
100000	0.841
1000000	0.858
10000000	0.872
100000000	0.883
1000000000	0.892
1E+10	0.900
1E+20	0.939
1E+50	0.970
1E+100	0.982
1E+200	0.990
inf	1.000

(Fig. 10.03)

This graph is used for comparing asymptotic expansions against the actual function. An example of this would be,

$$wzl(x) \sim \frac{x}{\log(x)} \qquad (10.08)$$

From this, one can quickly see that this asymptotic expansion is true.

CONCLUSION

For functions that model a rapid rise to a steady-state, such as bullet accelerating down the barrel of a gun, the Sklaric graph is very useful. The main drawback is that the function,

$$y = \frac{1}{wzl(1/x)} \qquad (10.09)$$

does not have "simple" properties like the function,

$$y = 1-e^{-x} \qquad (10.10)$$

Equation (10.10) is used in electronics to describe the charging of a circuit.

Equation (10.09) has infinite slope at x=0 which means (10.09) cannot be expanded in a Tayler series around x=0. This along with the invariance of scaling makes Sklar graphs less flexible than logarithmic graphs for most scientific work.

Sklar graphs are specialized graphs like Probibility graphs (for Normal Distribution) where they are used for specific applications. The best use for them to date is for bullet acceleration studies (chapter 12).

The second type of graph, based on Coupled Roots is most useful for numerical asymptotic study where numbers ranging from 1 to infinity need to be plotted.

10.01:

Professor Ronald Sklar was a Numerical Analysis professor at the State University of New York at Old Westbury. One of the authors studied under him from January 1981 to December 1982.

A name was needed to describe the 'fish-eye' graphs. The graph is based on the Wexzal function but is not the Wexzal function itself. To call the graphs 'Wexzalic' would have caused confusion as 'Wexzalic' here means "involving the Wexzal function". This chapter discusses two types of Wexzalic graphs.

References for Chapter #10

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(1) Milek, Bob "Barrel Length vs. Velocity"
From "Guns & Ammo" page 46

Chapter 11

Application of the Wexzal in Ballistics

INTRODUCTION

External ballistics concerns itself with the study of bullet flight from the time the bullet leaves the barrel of the gun until it reaches the target. Of major interest is the question of velocity decay. This is important to the shooter who wishes to know if the bullet has enough energy to destroy the target.

The measure of a bullet's resistance to velocity decay is called the "Ballistic Coefficient" and it is denoted by BC. It is a ratio of velocity decay between the bullet in question and a "standard" (reference) bullet. This standard bullet has a BC=1.0. The larger the BC, the more efficient (more velocity at target for a given muzzle velocity) the bullet is. The BC is a function of the bullet's weight, its shape (or form) and the air density where the measurment is taken. The following table give examples of some BC values:

Projectile (description)	BC	
0.177 cal pellet "Silver Sting"	0.0180	{11.01}
0.22 Long Rifle bullet	0.10	{11.02}
"Average Hunting bullets"	0.2 - 0.6	
8mm sS bullet (198 grain)	0.588	{11.03}
13mm T-Gewehr bullet (811 grains)	1.266	{11.04}

(Fig. 11.01)

In the U.S.A., the standard model is called the "G1" model. This was developed by the U.S. army around the time of World War I. Tables of this function can be found in most text books on ballistics [11.01]. The tables are of the form of v and G1(v) where 'v' is velocity. It is this model (and notation) that will be used in this chapter.

Discussion of a bullet's BC is done mostly by "serious" reloaders. Reloading manuals [11.02] not only give loading data (amount/type of powder, type of brass & primer and bullet shape/weight) but also the BC for each bullet made by that company. This aids the loader in determining downrange performance. Some manuals also contain tables containing the muzzle velocity, velocity at 100 yards, etc. An example is the following for a U.S. 30-06 "Accelerator" [11.03].

Yards	Velocity in ft/sec
0.0	4080.0
100.0	3485.0
200.0	2965.0
300.0	2502.0
400.0	2083.0
500.0	1709.0

(Fig. 11.02)

The question becomes: What type of function is the velocity decay? Is it exponential? Hyperbolic, or linear? Is there a "simple" way to calculate the BC when given data in the form of fig 11.02? Can the flight time be quickly computed? What does the trajectory look like? The rest of this chapter addresses these questions.

VELOCITY DECAY

In the book "Jagdballistik" (Ballistics for hunting) [11.04] velocity decay is given in the form of:

$$v = \frac{a}{e^{(b*x)}} \qquad (11.01)$$

where 'v' is velocity in metres/sec and x is in metres. The coefficients 'a' and 'b' are determined by curve-fitting. The larger 'b' is, the faster the velocity would decay. The 'a' coefficient is approximate to the muzzle velocity.

This simple formula has the advantage of being easy to integrate (to calculate flight time) and because it is an exponential, one can calculate the coefficients by first transforming the equation into the form of:

$$\ln(v) = \ln(a) - b*x \qquad (11.02)$$

Most scientific calculators have the ability to perform this type of curve-fit. (The calculator manuals might refer to this as "Linear Regression" as this is a statistical operation also). Using data from fig 11.02, we obtain,

$$v = 4142.2328 / e^{(1.73277E-3 * x)}, \quad RMS = 36.267 \qquad (11.03)$$

where 'v' is in ft/sec and 'x' is in yards. The Root Mean Square is just a little over 36 ft/sec.

A BETTER DESCRIPTION OF VELOCITY DECAY

Is there a model that better fits data like in fig 11.02? By "better" we mean having a lower RMS value. Using standard units let:

- a = "Asymptotic" velocity in ft/sec
- b = "Decay rate" in 1/ft
- x = Distance in feet
- v = Velocity in ft/sec
- t = Flight time in seconds
- k = Mass of bullet in slugs (32.2 lb = 1 slug)
- s = Scope height in feet
- d = Bullet drop in feet
- y = Height above line of sight in feet
- E = Firing angle in radians
- z = Distance to target in feet
- g = Acceleration due to gravity (32.2 ft/sec^2)

Using (11.01) as a basis (as the exponential has the "right idea") we write:

$$v = \frac{a}{} \qquad (11.04)$$

$$\mathrm{wzl}[e^{(b*x)}]$$

Why "wrap" a Wexzal around the exponential part? For non-negative functions, the Wexzal "distorts" that function. For the simple case of f(x)=x, the wexzal does the following:

$$\begin{aligned} \mathrm{wzl}(x) &> x \quad \text{for all } x \text{ in } [0,10) \\ \mathrm{wzl}(x) &= x \quad \text{at } x=10 \\ \mathrm{wzl}(x) &< x \quad \text{for all } x > 10 \end{aligned} \qquad (11.05)$$

This "bending" behavior has proven useful. Performing a non-linear curve-fit on fig. 11.02 using (11.04) leads to:

$$v = \frac{10205.916}{\mathrm{wzl}[e^{(1.01929E-3*x)}]}, \qquad RMS = 7.406 \qquad (11.06)$$

Note that the RMS is about 4.9 times smaller. The 'a' value is nearly equal to the muzzle velocity times wzl(1). This is a result of:

$$\mathrm{wzl}(e^0) = \mathrm{wzl}(1) = 2.506184146 \qquad (11.07)$$

Can we calculate the flight time of (11.04) in closed form?

COMPUTING FLIGHT TIME

Knowing the flight time from muzzle to target is useful in that this information aids in calculating the correct "lead-angle" to give a moving target. If aimed correctly, both the projectile and target will arrive at the exact same location at the exact same time.

As the velocity is given as a function of distance this will result in a differential equation that can hopefully be solved in closed form. Here we use the term "closed form" to mean one can write a formula involving Wexzals and (if need be) other known higher functions {11.05}. It is assumed that these functions are "easy" to calculate; no Runge-Kutta or Simpson's Rule needed. This reduces the need for computing power. For the shooter, this means that a programmable calculator is all that is needed to make the calculations; no powerful 486DX/33 laptop need be taken to the shooting range.

Writing (11.04) in differential form,

$$\frac{dx}{dt} = v = \frac{a}{\mathrm{wzl}[e^{(b*x)}]} \qquad (11.08)$$

leads to the integral,

$$t = \frac{1}{a} * \int \mathrm{wzl}[e^{(b*u)}]\, du \qquad (11.09)$$

Removing constants, we have the form of the integral,

$$\int \mathrm{wzl}[e^u]\, du = \int \frac{\mathrm{wzl}(z)}{z}\, dz \quad \text{where } u = \ln(z) \qquad (11.10)$$

```
    /               /
```

This integral can be written in closed form (see chapter 05) and the result is:

```
    /
    | wzl(z)
    | ------ dz = wzl(z) + ei{ln[wzl(z)]} + c                   (11.11)
    |   z
    /
```

From (11.11) {11.06} one obtains the flight time:

```
              b*x
            /e
        1    | wzl(u)
    t = --- * | ------ du                                        (11.12)
       a*b   |   u
            /1
```

If we define B(u) to be,

```
    B(u) = wzl(u) + ei{ln[wzl(u)]}                               (11.13)
```

then the flight time can be written in standard form as,

```
        1
    t = --- * {B[e^(b*x)] - B(1)}                                (11.14)
       a*b
```

where B(1) = 4.180218835...

Both B(u) and {B(e^u)-B(1)} will be tabulated in the appendix.

COMPUTING AVERAGE VELOCITY

There is not much need to know the average velocity if one can compute the flight time exactly with little effort. However, if the average velocity value was needed, it too can be computed in closed form. Using (11.04) we obtain the average velocity as follows,

```
                                   b*x
           /x                    /e
        a   |      du        a    |    dz
    v` = - * | ------------ = --- * | -------- ,   u=ln(z)      (11.15)
        x   | wzl[e^(b*u)]   b*x   | z*wzl(z)
           /0                    /1
```

The integral,

```
    /
    |    dz        1
    | -------- = ------ - ei{-ln[wzl(z)]} + c                   (11.16)
    | z*wzl(z)   wzl(z)
    /
```

can be written in closed form. So the average velocity, v`, is:

```
                 1
    Let P(u) = ------ - ei{-ln[wzl(u)]}                         (11.17)
              wzl(u)
```

$$v` = \frac{a}{b*x} * \{P(1) - P[e^{(b*x)}]\} \quad (11.18)$$

where P(1) = 0.6508866537...

COMPUTING DRAG FORCE ON BULLET

Once the bullet leaves the muzzle, it would be interesting to know how much air resistance the bullet experiences. This can be calculated as follows,

$$\text{acceleration} = \frac{dv}{dt} \quad (11.19)$$

Using (11.04) we have,

$$dv = \frac{-a*b * e^{(b*x)}}{wzl[e^{(b*x)}]^2*\{m + \frac{e^{(b*x)}}{wzl[e^{(b*x)}]}\}} dx \quad (11.20)$$

but dx = v * dt so we obtain as final result for the acceleration,

$$\frac{dv}{dt} = \frac{-a^2 * b * e^{(b*x)}}{wzl[e^{(b*x)}]^3*\{m + \frac{e^{(b*x)}}{wzl[e^{(b*x)}]}\}} \quad (11.21)$$

The drag force in pounds is then,

$$f = k * \frac{dv}{dt} \quad (11.22)$$

For a 198 grain bullet (8.78E-4 slugs) with BC=0.5 and a muzzle velocity of 2600 ft/sec experiences a drag of 1.3342 pounds on muzzle exit. The acceleration is -1518.8 ft/sec^2 or about 47 G's. No wonder some "cheap" bullets blow-up after exiting the muzzle! {11.07}

VELOCITY & DISTANCE AS A FUNCTION OF TIME

Upto this point the velocity and acceleration have been expressed as a function of distance. It would be useful if these equations could be written as a function of time as that is the way most mathematical models are written. Let us define,

$$x=B(y), \quad y=invB(x) \quad (11.23)$$

where B(y) is the form given in (11.13). An interesting property of invB(x) is that,

$$\frac{dy}{dx} = \log\{wzl[invB(x)]\} = \log[wzl(y)] = \frac{y}{wzl(y)} \quad (11.24)$$

This result is obtained using the "derivative of inverse function" rule. What makes this interesting is that (when constants are stripped) the velocity decay problem boils down to a first degree differential equation involving the logarithm of the Wexzal. Taking equations (11.04) and (11.12) and rearranging to get them to be a function of time we get:
For velocity,

$$v = \frac{a}{\mathrm{wzl}\{\mathrm{invB}[a*b*t+B(1)]\}} \qquad (11.25)$$

For distance,

$$x = \frac{1}{b} * \ln\{\mathrm{invB}[a*b*t+B(1)]\} \qquad (11.26)$$

The computation of invB(x) involves iteration. For quick "field" calculations, a series solution of (11.24) when given y(0)=1 (exact solution: y=invB[x+B(1)]) is,

$$y = 1 + 0.399*x + 0.041488*x^2 - 0.0011434*x^3 + \ldots \qquad (11.27)$$

This can be used to get approximations for small values of x. Using (11.27) in (11.25) & (11.26), require that the term a*b*t be small.

HOW DOES THIS MODEL COMPARE TO "REAL-WORLD" DATA?

Equations (11.04), (11.14) and (11.18) are very interesting to look at but how close are they at describing reality? For what velocity range and/or BC range are they valid for? From theorems from calculus, if the velocity equation is correct, then the flight-time equation must be correct also (provided the integral as stated is correct!). Because of this, all that needs to be verified is the velocity (11.04) equation. This was done as follows,

One of the authors used Vol II of [11.02] which gives velocity decay data along with the BC for every bullet that company makes. The data for bullets having muzzle velocities ranging from 1000 to 4000+ ft/sec and BC values ranging from 0.11 to 0.620 were entered into a laptop computer. A curve-fit "contest" was held comparing the exponential decay model (11.01) against the Wexzalic-exponential decay model (11.04). A FORTRAN program that calculated bullet flight via the G1 model was used also to generate data. This was used for extreme cases like a bullet having a muzzle velocity of 4000 ft/sec with a BC=0.0180 which would describe an airgun pellet. The data from [11.02] and the G1 program were assumed to be "exact" i.e. the tabulated data was not curve-fitted with some unknown formula that would cause the tables to be biased toward one form (11.01 or 11.04) over another. As a further test, German tables from RWS (Rheinische-Westfaelische Sprengstoff - Rheinland Explosive Works) and "Waffen Revue" were used.
The result of all of this calculation?

(1) The Wexzalic model provided a better fit provided that the muzzle velocity and impact velocity were >= 1370 ft/sec.

(2) The value of the ballistic coefficient made no difference on the outcome of which model was better. Only that the velocity made a difference; not the change in velocity (as dictated by the BC).

Why the "break" at 1370 ft/sec? We know from aerodynamics that the transonic range (~900 - 1300 ft/sec at sea level) produce great changes in drag. This is caused by changes in type of airflow around the body (airplane, bullet, etc) travelling thru the air. Another thing to consider is the following:

```
DECLARE FUNCTION bigg# (x#)
DECLARE FUNCTION g1# (x#)
DEFDBL A-Z
DEF fnlgt (x) = LOG(x) / LOG(10#)
CLS
CLEAR
v = 2600#
g = .588#
FOR y = 0# TO 1000#
f = y * 3#
e = -f / g
t = 100#
w = v - t
h = 10# ^ 99
50 GOSUB 1000
j = ABS(c - e)
IF j >= h THEN
        GOTO 210
        END IF
h = j
w = w - t
IF w > 0# THEN
        GOTO 50
        END IF
210 w = w + t
FOR j = 1 TO 10
GOSUB 1000
p = w
w = w - (c - e) * g1(w)
NEXT j
PRINT USING "   #####.##"; y; w
NEXT y
END

1000 a = fnlgt(v)
b = fnlgt(w)
k = 8
n = k * FIX(1# + ABS(b - a))
d = (b - a) / n
u = (n - 2) / 2
s = bigg(a) + bigg(b): a = a + d: s = s + 4# * bigg(a)
FOR q = 1 TO u: a = a + d: s = s + 2# * bigg(a)
a = a + d: s = s + 4# * bigg(a)
NEXT q
c = s * d / 3#
RETURN

FUNCTION bigg# (x#)
DEFDBL A-Z
p = 10# ^ x
bigg = LOG(10#) * p / g1(p)
END FUNCTION

FUNCTION g1# (x#)
```

```
DEFDBL A-Z
IF x >= 2600# THEN
        GOTO 100
        END IF
IF x >= 1800# THEN
        GOTO 200
        END IF
IF x >= 1370# THEN
        GOTO 300
        END IF
IF x >= 1230# THEN
        GOTO 400
        END IF
IF x >= 970# THEN
        GOTO 500
        END IF
IF x >= 790# THEN
        GOTO 600
        END IF
r = 10# ^ (5.66989 - 10#) * x
GOTO 900

100 r = 10# ^ (7.60905 - 10#) * x ^ .55#: GOTO 900
200 r = 10# ^ (7.0962 - 10#) * x ^ .7#: GOTO 900
300 r = 10# ^ (6.11926 - 10#) * x: GOTO 900
400 r = 10# ^ (2.9809 - 10#) * x ^ 2: GOTO 900
500 r = 10# ^ (6.80187 - 20#) * x ^ 4: GOTO 900
600 r = 10# ^ (2.77344 - 10#) * x ^ 2: GOTO 900
900 g1 = r
END FUNCTION

-----------------------------------------------------------
                 (Fig. 11.03)
```

Fig. 11.03 is a BASIC (actually QBASIC in DOS 5.0) program that uses the G1 model to compute the impact velocity when given muzzle velocity, range to target and the BC. In the function G1(x) where the "IF" statements are, the velocity ranges that are tested are >=2600 ft/sec, >=1800 ft/sec, >=1370 ft/sec, etc. Note the corresponding "R=" statements after the numbered program labels (100,200,300 etc).
The equations are in the form of r=k*x^y where k and y are constants.
The values for k written in the form of 10^(x.xxxx-10) is a very traditional way of writing antilogarithms of negative numbers. The three equations labelled (100,200,300) have y values of (0.55,0.70,1.0)
The first two equations are convex functions like the Wexzal. A convex function is one where if one were to draw a straight line from (x1,y1) to (x2,y2), where {(x1,y1),((x2,y2)} are points on f(x), the line would lie under the function curve. We think this property accounts for the excellent agreement with the Wexzalic model. All we are certain of is the Wexzalic model gives closer agreement with the G1 model for high velocity bullets. The exponential model does much better than the Wexzalic model for velocities less than 1370 ft/sec.

OBTAINING BC VALUE FROM VELOCITY DATA

For velocities >=1370 ft/sec, how does one obtain the BC from a set of (distance,velocity) reading? This is important as shooters use the BC value in judging bullets for their use in hunting, target shooting etc.

One of the authors discovered that the BC is a linear function of the muzzle velocity (V0) in ft/sec and the 'b' coefficient in (11.04). This was found as follows: A FORTRAN program was written that would generate, via the G1 model, tables for varying initial velocities and BC. This was as follows (in pseudo-code)

```
For V0=2000 to 4000 step 10
For BC=0.1 to 1.3 step 0.1
For Yards=0 to when_ever_computed_vel_got_<_1370 step 5
Compute velocity via G1 model using (BC,V0,Yards)
Store in array the values (Yards,velocity)
next Yards
Compute 'a','b' coeffs in (11.04) from this data and
        store array(BC,1/b)
next BC
Now compute a linear fit in form of y`=b_coef * x` + a_coef
        using (BC,1/b)
Write to external file the values (V0,b_coef)
next V0
end
```

The values in the external are then linear curve-fitted to give the final form of the formula. The reason for using (BC,1/b) instead of (BC,b) is because we wanted the relationship to be monotonic increasing (as BC goes up, so does 1/b).

This program was run on a 486DX/33 laptop using a 32-bit FORTRAN. The runtime was over 6 hours. The final formula is:

$$BC = \frac{1/b}{0.4705469931 * V0 + 3038.91861} \qquad (11.28)$$

This formula has been checked against tables and has been found to be at most 0.003 off from the actual BC value. This testing was done by selecting data that had a known BC and velocity values for 0,100,200 etc yards. From this the 'a','b' coefficients from (11.04) were computed. The BC was then computed from this and compared to the actual BC.

CONSTRUCTING A TRAJECTORY

The basic components of a trajectory need to be defined before the formulae for the path of a bullet can be derived. When one looks down the sights of a gun (or telescope if the gun has one) at the target, this is called the "line of sight". The shooter makes the assumption that the projectile will travel in a straight line to the target; much like the path a laser beam would take. In reality, the path the projectile would take is parabolic-like (not an exact parabola due to wind resistance). On most guns, a telescope sight is about 1.5 inches above the barrel. For guns with "iron" sights (two metal "leaves" mounted on the barrel with notches on them) the sights are about 0.9 inches above the barrel. From this, one can see that the target is below the line of sight. To correct this, the barrel (relative to the line of sight) is pitched up at a small angle. This angle is adjustable by the shooter and that is what he does when he adjusts the sights so the gun (for a given bullet) will hit the target at a specified distance. Shooters call this "zeroing" their guns.

When a gun is fired, the bullet leaves the barrel and after a short distance, rises above the line of sight. At some point, the bullet will reach a maximum height above the line of sight. This is called the "maximum ordinate" or "max ord" for short. This occurs at about 55% of the distance to the target. After reaching max ord, the bullet starts to drop. It will in due time, cross the line of sight. The distance at which this occurs is called the "zero". If the shooter has adjusted his sights correctly, the zero should occur at the target. In reality, there are two zeros; first one is when the bullet crosses line of sight on its way to max ord; the second on the descent to the target. Most of the time, it is the second zero that is of interest. The first is useful to know in that the shooter knows that for targets located at distances within the two zeros, he needs to aim low. For a target either closer than the first zero or further than the second zero, he needs to aim high.

All of this gives the impression that for a given sight setting, the gun can only be used on targets located at the zero. This is true only in shooting competition where the targets are at a fixed distance and an exact location (the "bullseye" - the black center of a paper target). For field use (hunting and military) the idea of "point blank range" comes into use.

The informal definition of point blank range is a distance that is so short that a gun does not need to be carefully aimed to hit the target. This is seen in newspaper stories like "The policeman shot the bank robber at POINT BLANK RANGE after the robber attempted to get away". For our use, this definition is not correct.

Point Blank Range (PBR) is that distance such that the bullet has dropped max ord distance BELOW line of sight. It is clear that this occurs after the second zero. PBR is a function of max ord which in turn is a function of the sight setting and bullet characteristics. Why is PBR important?

Most field targets (game animals, enemy solders, etc.) have a circular area where they can be hit and still be destroyed. It is the size of this area that determines (along with impact energy) the maximum range the target can be destroyed. For example, a deer (the larger North American type) has a vital area that is about 6 inches in diameter. Using the center of this, we have upto 3 inches above and 3 inches below in which to score a "kill". If a hunter has a gun that is adjusted so it gives 3 inches max ord, the deer can be located any distance from 0 feet from the gun upto the gun's PBR, and all the hunter need do is aim at the center of the deer's vital spot to score a kill. He does not need to make any adjustments or compensate in any way; all he does is "point & click". This is important in that in field conditions, one does not know the exact distance to the target. All one can do is estimate if the target is within PBR.

Armies do the same thing. Most major armies during WWI and WWII had their battle rifles configured with iron sights that started at 200 yards or more. The American Springfield had a leaf fold-down sight that started at 100 yards and could be adjusted to over 1000 yards. This same sight could be folded down to give a "default" setting of ~400 yards. The max ord was about 12 inches. The Germans with their Gewehr 98 had an elegant "rollercoster" sight that started at 400 metres. This gives a max ord of about 12 inches also. The thinking in 1914 in both the German High Command and the American War Department (now called DoD) was that long range shooting (~400 yards) was "the answer". Remember, 30 years before, armies had black powder arms and the upper effective range was about 200 yards. Part of the lesson learned from WWI by the Germans was that most accurate shooting occur at ~100-200 yards so they altered their Gewehr 98's to accept 100 metre sights. By WWII sniping became "popular" so more precise sighting was in demand by all sides.

How can we sight a rifle in at a standard 100 yard range so we can obtain the desired trajectory?

BULLET DROP

One has heard of the quasi-correct statement that a bullet fired from a level gun and another bullet dropped from the shooter's hand fall at the same rate. In a vacuum this would be true but in reality, it is not so.

One has observed at an American football game the flight of a football. In a long pass (> 50 yards) the football appears to hang in the air for an overly long period of time. This is called "hang time". This is due to angle of attack (angle football makes with the air in its flight path), increased lift, etc. Bullets do the same thing on a smaller scale.

The book "Hatcher's Notebook" [11.05] on page 627 contains a drop table. This table gives the drop has a function of the velocity ratio (V/V0) and flight time. Curve fitting this table gives,

$$d = drop\_in\_ft = 16.1 * (v/v0)^{0.3} * t^{2} \qquad (11.29)$$

The flight time is given in (11.14) and the velocity in (11.04) so drop as a function of distance is given by,

$$d = \frac{0.5*g*wzl(1)^{0.3}}{wzl(e^{b*x})^{0.3}} * \left[\frac{1}{a*b} * [B(e^{b*x}) - B(1)] \right]^{2} \qquad (11.30)$$

Hatcher's table is valid for v/v0 >= 0.333333333 which is good for Wexzalic use where v,v0 must be >= 1370. This ratio would mean that for an impact velocity of 1370 ft/sec, v0=1370*3=4110 ft/sec. Hatcher's book is an excellent reference for other technical information involving guns.

If we take equation (11.14) and solve it for x and substitute in (11.30) and then expand in a Taylor series we get,

$$d(t) = 16.1*t^2 + 0.9202*a*b*t^3 + 0.0645*(a*b)^2*t^4 + \ldots \qquad (11.31)$$

In a vacuum, the ballistic coefficient is infinite. Using equation (11.28) we get b=0 for an infinite ballistic coefficient. In this case, (11.31) reduces to the classical,

$$d(t) = 16.1*t^2 \quad \text{when } b=0 \qquad (11.32)$$

Equation (11.31) is used to verify that the Wexzalic equation reduces to the classical case when the ballistic coefficient goes to infinity.

From all of this, we can construct the trajectory equation. Classical texts give,

$$X(t) = V0*t*\cos(E) \qquad (11.33)$$
$$Y(t) = V0*t*\sin(E)-0.5*g*t^2 \qquad (11.34)$$

We use this to derive our trajectory equation. The second term in (11.34) is the drop term. Solving (11.33) for t and substituting into (11.34) gives a formula in the form of Y=f(X). We use this idea to produce,

$$y = x*\tan(E) - d(x) - s \qquad (11.35)$$

where s = scope height,
d(x) = drop as fcn of x,
E = firing elevation,
x = distance in x direction,
y = height above line of sight.

One can use (11.35) to make a trajectory table if the firing angle is known along with the muzzle velocity and ballistic coefficient. Most of the time the firing angle is not known. We set x=z, where z is distance to target in feet and solve (11.35) for E when y=0. Doing this we obtain,

$$E = \arctan\left(\frac{d(z) + s}{z}\right) \qquad (11.36)$$

As an example, take a German Gewehr 98. It fires a 154 grain 8mm bullet having a BC=0.353 at a muzzle velocity of 2936 ft/sec. The lowest setting of the iron sights is for 400 metres. What is the firing angle? We have,

```
V0=2936 ft/sec
BC=0.353
s=0.9 inches = 0.075 feet
z=400 metres = 1312.34 feet
```

From this we generate,

```
a = V0*wzl(1) = 7358.157
b = 1/[BC*(0.471*V0+3039)] = 6.406498E-4
v @ z = 1880.62185 ft/sec
t @ z = 0.55928 seconds
d(z) = 0.5*g*(v/V0)^0.3*t^2 = 4.406038 feet
tan(E) = (d(z)+s)/z = 3.41454E-3
E = 3.414527E-3 radians = 0.19564 degrees = 11.74 minutes.
```

The barrel is then pitched up just under 1/5 of a degree.

SIGHTING IN A RIFLE AT A STANDARD RANGE

In the United States and many countries of Europe, outdoor shooting ranges are common. Most ranges (for rifle use) have paper targets starting at 100 yards and ranging to as much as 1 mile (1760 yards). For a standard 100 yard range, the question of how to zero a rifle for a range other than 100 yards is raised. Most hunters set their rifles to have a zero between 150 and 350 yards depending on the calibre of the rifle, bullet weight selected and type of game animal hunted. Since the zero selected by the hunter is (most of the time) greater than 100 yards, the bullet will hit "high" (above the bulleye) on the 100 yard target. The question is: "How much?".

To sight in a rifle for a specified zero using a standard 100 yard target one does the following:

(1) Obtain the V0 and BC of selected bullet.
(2) Using specified zero, perform the calculations above to obtain the firing angle as this is needed.
(3) Use equation (11.35) with x=300 to obtain the impact point on the target.

So for the Mauser example above, we need to compute the velocity and drop for the bullet at 300 feet. Using (11.04) and (11.14) we obtain,

```
v @ 300 = 2671.534
d(300) = 0.179571
y=x*tan(E)-d(x)-s = 300*3.41454E-3 - 0.179571 - 0.075 = 0.769791 ft
```

which is about 9.24 inches high.

CONCLUSION

This chapter outlined some ballistic formulas involving the Wexzal function. Experimental evidence indicates that for very high velocites, these formulas better model actual bullet behavior then classical methods. This indicates that there is second (and maybe third) order effects that are accounted for by the Wexzal function.

This model has the advantage of being easy to implement on a programmable calculator or small laptop computer. The shooter then can quickly make ballistic calculations in the field without having to consult complex ballistic tables. By using (11.28) one can make a distance/velocity table when given the BC and initial velocity. Solving for b in (11.28) gives,

$$b = \frac{1}{BC * (0.4705469931 * V0 + 3038.91861)} \qquad (11.37)$$

One can approximate a as,

$$a = V0 * wzl(1) = 2.506184146 * V0 \qquad (11.38)$$

From these two coefficients a table can be made.

11.01:

Low ballistic coefficients for pellets is a blessing and a bane for airgun shooting. Airgun shooting has been a "serious" activity for Europeans for many years due to tight living space and tight firearm laws. In the U.S.A. airgun shooting has been viewed as something for children to do until old enough for "real" guns. Due to tighter gun laws in the U.S. coupled with the efforts of Dr. Robert Beeman to bring European airguns to the U.S., airgun shooting has become an "adult" sport. "Magnum-mania" - that is, the desire for more power has fueled the popularity of airguns. Many airguns experts date the start of the "magnum-craze" from either 1978 with the introduction of the Feinwerkbau 124 (800 ft/sec) or in 1981 with the introduction of the Beeman R1 (~1000 ft/sec). Today there are many 1000+ ft/sec guns.

There are now airguns (0.177 calibre) that shoot over 1100 ft/sec. These are made by Hermann Weihrauch AG and RWS. Both firms are in Germany. Even with this high velocity, these guns have a maximum effective range of about 50 yards (150 feet) due to the rapid decrease in pellet velocity. The rule-of-thumb is that a pellet halves its velocity every 150 feet. Instructions that come with these guns state that the gun is dangerous out to 400 yards. No need to worry about the pellet travelling many miles and hitting anyone. Because of the ballistic properties of pellets, pellet rifles are outstanding for popping backyard pests with next to no noise and very little hazard. The beloved American 0.22 LR calibre rifle has no fear of being driven into extinction by airguns. An average pellet weighs 8 grains; the 0.22 weighs 40 grains.

11.02:

The 0.22 LR (Long Rifle) is for many Americans their introduction to "real" guns. It fires a 40 grain lead bullet at about 1200 ft/sec. Because of the higher ballistic coefficient (than pellets) every 50 round box (or 500 round "brick") contain the warning that the bullets can travel over 1.5 miles. The 0.22 is popular due to nearly no recoil, low cost and posessing a sharp "crack!" as a report instead of an earth-shaking "boom!" everytime its fired.

11.03:

The Germans in WWI used a 154 grain bullet as their standard round in their Gewehr 98 Rifles. In the 1920's the Reichswehr (army) decided to make the 198 grain round standard in both machine-guns and rifles. They found that the 154 grain round was too "light" for long range shooting. The heavier 198 grain round had a lower muzzle velocity but much better downrange (~400 yards) performance. This made logistics much easier also. This was the offical German round in WWII.

11.04:

In 1916, during WWI the Germans faced a new problem. The British were the first to field battle tanks. The Germans countered with a large bolt action rifle that shot a 13mm (~0.51 calibre), 811 grain bullet at 2600 ft/sec. This gun, called the "Tankgewehr", had a 40 in barrel and was first put into service in 1918. The German gun magazine "Waffen Revue Nr. 83 IV Quartal 1991" (Weapon Review #83 4th quarter) has an article starting on page 37 titled "Das 13mm Tankgewehr von Mauser im Ersten Weltkrieg" (The 13mm Tankrifle from Mauser in WWI). It contains many photos showing the size of this gun. A K98 is also shown for comparsion.

11.05:

Don't be intimedated by the term "higher function". Higher functions are (most of the time) series solutions to differential equations. These solutions are named after famous mathematicians who first worked on the problem. The problem became important enough that the solution got tabulated and named. An example of this is the Bessel function. It is a solution to a second order differential equation that describes star motion. A German mathematician first solved this equation (in series). This series became important that instead of writing out the series, mathematicians use the notation Jn(x). Mathematical handbooks contain tables and formulae involving higher functions.

Likewise, the Wexzal is a higher function. Instead of saying "the solution is a function of the inverse of y*log(y)", we would say "the solution is a function of the Wexzal." There are many higher functions. Some are famous and others are not well known.

The function ei(x) is called the exponential integral and it also is a higher function. It is defined to be

$$ei(x) = \int_{-inf}^{x} \frac{e^u}{u}\,du$$

Ei(x) appears in many Wexzalic integrals.

11.06:

The integral,

$$\int \frac{wzl(u)}{u}\,du = wzl(u) + ei\{\ln[wzl(u)]\} + c$$

was first calculated in closed form in April 1983. This was to solve a problem involving the asymptotic expansion of,

$$\frac{d}{dx}\,wzl(x) = \frac{1}{m + \frac{x}{wzl(x)}} \sim \frac{wzl(x)}{x} \sim \frac{1}{\log(x)}$$

This is clearly not related to guns! The Wexzal velocity decay theory presented in this chapter was developed in March of 1993; nearly ten years after the calculation of the flight time integral. Another integral calculated in the same era is,

$$\int \frac{dx}{x*wzl(x)} = \frac{1}{wzl(x)} - ei\{-\ln[wzl(x)]\} + c$$

This answered the question of convergence of integrals of this type as in,

```
    /inf
    |    dx
    | -------- = 0.6508866537...
    | x*wzl(x)
    /1
```

Today, this integral is used to compute average velocity.

11.07:

Bullet blow-up is caused more by the rotational acceleration then the sudden translational deacceleration. Once the bullet leaves the muzzle, the rotational forces cause the bullet to tear itself apart. The main way the shooter knows that this has occured is when there is no hole anywhere on the paper target and he knows the sights are set correctly. Bullets designed for hunting, if overdriven (shot at higher velocity than recommended by bullet company), can sometimes explode just after leaving the muzzle. The reason for this is that the hunting bullet must be able to expand freely just the right amount upon hitting the game animal but yet be strong enough to hold itself together during it's journey to the target. This conflicting set of requirements is what keeps the R&D department of major bullet companies busy.

Full Metal Jacket (FMJ) bullets (like what armies use) do not seem to suffer the problem of bullet explosion as they have a one piece metal "jacket" covering the entire bullet so no lead shows. Because of this FMJ bullets do not expand on impact and thus are not very useful for hunting. They are good for use at target practice. In the U.S.A. surplus military ammo can be had for as little as 25% the cost of hunting ammo.

References for Chapter #11

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Chapter 12

Application of the Wexzal in Firearms

INTRODUCTION

In the field of firearms comes the question of barrel length vs. muzzle velocity [12.01]. Most shooters are aware of the fact that everything else held constant, longer barrels (within practical limits) produce higher muzzle velocities. Studies have been made by both hobbyists and experts on this topic. The muzzle velocity of a gun with a long barrel is measured and then the barrel is cut one or two inches and then remeasured. This is kept up until a table of 6-7 readings is made. (Fig 12.01)

Table for 7mm Mauser M1893 with 140 gr. bullet. [12.02]

Inches of barrel	Muzzle Velocity in ft/sec
18	2506
20	2561
22	2608
24	2650
26	2687
28	2721
30	2752

(Fig. 12.01)

LOGARITHMIC VS. WEXZALIC CURVE-FIT

From initial inspection of Fig. 12.01, the trend is logarithmic. If we perform a logarithmic curve-fit, we find,

$$\text{Muzzle\_velocity} = 1105.616 * \log(\text{bbl\_length}) + 1121.568 \qquad (12.01)$$

where "bbl_length" is the barrel length in inches.

The Root-Mean-Square (RMS) of this fit is 2.146 ft/sec. For a data-set whose average value is 2641, this leads to an average error of 0.08%. Observing that the curve tends to flatten as the barrel length increases and to further impose the condition that at zero length, the muzzle velocity be set to zero, we try a curve-fit in the form of,

$$\text{Muzzle\_velocity} = \frac{a}{\text{wzl}(b/\text{bbl\_length})} \qquad a,b>0 \qquad (12.02)$$

Where 'a' and 'b' are constants. Because 1/wzl(1/x) ~ 1-1/(m*x)+... The constant 'a' tells the theoretic asymptotic velocity if given a barrel of infinite length. The constant 'b' tells how quickly the bullet accelerates up the barrel. The lower the value of 'b', the quicker the acceleration. If b=0, then we would have a constant function because wzl(0)=1. For b>0 and zero barrel length, we would have v=a/wzl(b/0) = a/inf = 0.

For curve-fitting, (12.02) cannot transformed into linear form. Therefore a non-linear curve-fitting method must be used. Most non-linear curve-fitting methods require an initial value ("guess") for all of the coefficients. So fitting the data in Fig 12.01 via a FORTRAN program called SKRFIT to perform non-linear curve-fitting in the form of (12.02) we obtain the values for 'a' and 'b',

$$\text{Muzzle\_velocity} = \frac{3294.530}{\text{wzl}(2.816/\text{bbl\_length})} \qquad \text{RMS}=0.8543 \qquad (12.03)$$

Here the RMS is 2.5 times smaller which means it is a better fit.

ENERGY & PEAK PRESSURE

From an equation in the form of (12.02) can we deduce the firing time, the peak pressure and location of peak pressure? Using standard engineering units let:

a = Asymptotic velocity in ft/sec
b = "Charging rate" in ft
L = Length of barrel in ft
x = Bullet location in barrel in ft
v = Velocity at point x in ft/sec
k = Mass of bullet in slugs (1 slug = 225400 grains)
r = Area of bore in square inches
f = Force in pounds
p = Pressure at point x in lb/in^2
E = Energy in ft-lb
t = Time in seconds.

From (12.02) let us calculate the acceleration & force on a bullet.

$$v = \frac{a}{\text{wzl}(b/x)} \qquad x \text{ in } [0,L] \qquad (12.04)$$

$$\frac{dv}{dx} = \frac{a*b}{x^2*\text{wzl}(b/x)^2*\{m+\log[\text{wzl}(b/x)]\}} \qquad (\text{ft/sec})/\text{ft} = 1/\text{ft} \quad (12.05)$$

Multiply (12.05) by v(x) to get acceleration.

$$\frac{dv}{dt} = v * \frac{dv}{dx} = \frac{a^2*b}{x^2*\text{wzl}(b/x)^3*\{m+\log[\text{wzl}(b/x)]\}} \qquad \text{ft/sec}^2 \quad (12.06)$$

From classical physics,

$$f = k * \frac{dv}{dt}, \qquad p = \frac{k}{r} * \frac{dv}{dt}$$

So the pressure on the bullet would be,

$$\frac{k}{r} * \frac{dv}{dt} = \frac{a^2*b*k/r}{\qquad} \qquad \text{lb/in}^2 \qquad (12.07)$$

```
  r   dt   x^2*wzl(b/x)^3*{m+log[wzl(b/x)]}
```

The area under (12.06) times the bullet mass is the energy of the projectile.

$$\text{Energy} = E = \int_0^x \frac{a^2 b k}{u^2\, wzl(b/u)^3 \{m+\log[wzl(b/u)]\}}\, du = 0.5\, k\, v(x)^2 \qquad (12.08)$$

When (12.07) is graphed from x=0 to L, the curve reaches a peak then quickly decays to zero. By solving the equation,

$$\frac{d}{dx}\left(\frac{dv}{dt}\right) = 0 \qquad (12.09)$$

numerically for x when given various values of b, one finds that there is a linear correlation between b and the location of the peak pressure, Xp. Xp is the value of x that satisfies (12.09) for a given value of b.

$$Xp = \frac{b}{m\,\text{sqrt}(2)\,\exp[\text{sqrt}(2)]} = b \cdot .39584 \qquad (12.10)$$

A handloader is someone who assembles their own ammunition. Handloading is a popular hobby in the United States and other couuntries that allow citizens to own and use firearms. For modern arms there are only four components that the handloader need address: Bullet, primer (small metal blasting cap that is inserted into the base of the shell), the shell (case) and the bullet. The selection of primer and case is based on the calibre of the gun itself. Different calibers are *NOT* interchangable even if they use the same bullet size. E.g. The American .30-06 and .30-30 use bullets that are 0.308 in diameter. The cases are different in shape and size. The handloader main focus is on the weight/style of bullet and the type and amount of powder. For a given calibre, the bullet style can range from flat nose to roundnose up to Spitzer (pointed). Weights can vary over a factor of 2. E.g. The .30-06 can be loaded with bullets weights from 100 grains up thru 250 grains. Different weight bullets (most of the time) require different powders. Powders vary in their burn rates. Contrary to popular belief, gunpowder does not "explode" (unless enclosed in a non-expanding enclosed volume). Gunpowder burns at a rate that is determined by the chemical properties of the powder. Each gunpowder company has their own line of powders that vary in burning rate. The numbers employed (e.g. IMR-4895) do not appear to the authors to correlate to burnrate; they are used as identifiers only.

When shooters speak of "fast" or "slow" gun-powders, they mean how quickly the powder reaches peak pressure with repect to bullet travel. A "fast" powder would have a smaller Xp value than a "slow" powder. Fast powers are used in pistols and smaller calibre rifles where rapid bullet acceleration is required (because the barrel is short). Longer barrelled arms and/or "magnum" arms are better served with slower powers. Shooters are also concerned with not only the "quickness" of the powder, but also the value of peak pressure. Most modern rifles operate in the range from 40000 to 65000 lb/in^2. The handloader must be careful of this fact when developing his loads. Exceeding the maximum pressure for a given load could cause the firearm to explode and injure (or kill) the shooter. Most reloading manuals have charts and tables explaining the maximum amount of powder (of a selected type e.g. IMR-4064) that can be safely used with a given bullet type/weight and primer.

Advice to reader: If the reader is interested to get started in handloading, please obtain expert instruction from a certified gunsmith or shooting instructor. Read all manuals and understand everything before proceeding. Handloading is in of itself quite safe if done correctly.

Handloading is most important to those whose guns are not longer "supported". E.g. Shooters who have Japanese Arasakas (7.7 mm) from WWII. No U.S. ammunition company currently makes ammunition for this arm. Current arms such as the AR-15 (M-16 lookalike) which uses .223 Remington ammunition do not suffer this problem.

Why is the location of peak pressure important? In this model, the sooner the peak pressure occurs (with respect to bullet travel) the quicker the velocity increase "flattens out". The shooter's rule-of-thumb is that (on the average) peak pressure is reached within one bullet length of travel. For an example of this discussion is the following:

A shooter has a German M1898 ("K-98 Mauser") with a 24-in barrel. He has two different powders (for a given bullet weight) that both produce a muzzle velocity of 2857 ft/sec [12.03]. One loading is a with a "fast" powder; the other with a "slow" powder. Being a fan of Mauser weapons, our shooter considers aquiring a WWI German M1898 Mauser with a 29.3-inch barrel with high hopes of obtaining higher muzzle velocities. Before shelling out the cash for such a costly item, he takes his K-98 and loads to a gun testing lab to obtain a time-history of powder pressure. [12.04] At the gun lab, scientists attach special pressure measuring equipment to the gun. An oscilloscope is used to obtain a time/pressure curve. Via standard curve fitting methods or cubic splines, it is possible to find the location of peak pressure. Out of this calculation, our friend learns that the fast powder has a value of Xp=0.06 and the slow powder has a value of Xp=0.08 He then starts to calculate:

```
-------------------------------------------------------|
| Mauser K-98:                                          |
| Bullet mass = k = 154 grains = 6.83E-4 slugs          |
| Barrel area = r = .323^2 * PI/4 = 0.08194 in^2        |
| b_fast = 0.06 / 0.39584 = 0.15158                     |
| b_slow = 0.08 / 0.39584 = 0.20210                     |
| a_fast = 2857 * wzl(0.15158/2) = 3319.95              |
| a_slow = 2857 * wzl(0.20210/2) = 3461.83              |
| L = 29.3 / 12 = 2.4417                                |
| v = a / wzl(b/L)                                      |
| v_fast = 3319.95/wzl(0.15158/2.4417) = 2926.86 ft/sec |
| v_slow = 3461.83/wzl(0.20210/2.4417) = 2943.87 ft/sec |
| Peak_pressure_fast = 53033 lb/in^2   [using (12.07)]  |
| Peak_pressure_slow = 43248 lb/in^2                    |
|-------------------------------------------------------|
                 (Fig. 12.02)
```

He gains from 70 to 90 ft/sec muzzle velocity depending on the type of powder used. In shooting, this is considered a good increase. {12.01}

FIRING TIME

Energy and velocity are both given as a function of distance, x, down the barrel. Up to this point time has not been discussed. It will be shown that this model can determine the time from peak pressure to muzzle exit; but is unable to determine the time from x=0 to peak pressure. Computing the reciprocal of (12.02) gives

$$\frac{1}{v} = \frac{wzl(b/x)}{a} \text{ sec/ft} \qquad (12.11)$$

So by integrating with respect to distance will give time.

$$\text{Firing\_time} = \frac{1}{a} * \int_{\text{Starting\_location}}^{\text{Bullet\_exit}} wzl(b/u)\, du \ \text{sec} \qquad (12.12)$$

The integral of wzl(1/x) can be written in closed form.

$$\int wzl(1/x)\, dx = \frac{1}{m} * \{1/\ln[wzl(1/x)] - \ln[\ln(wzl(1/x))]\} + c \qquad (12.13)$$

To see if we can compute the time from x=0 to bullet exit, we need to see if the integral is convergent at zero. (Fig 12.03)

Let us compute the limit,

$$\lim_{x \to 0+} \{1/\ln[wzl(1/x)] - \ln[\ln(wzl(1/x))]\} =$$

$$\{1/\ln[wzl(x)] - \ln[\ln(wzl(x))]\} \sim -\ln\{\ln[wzl(x)]\} = -\text{inf}$$

Value of x	Value of (12.13)
3.00	6.538161728
2.00	4.818573365
1.00	2.701279626
0.50	1.229903476
0.2239497283	0.0
0.10	-0.920430676
0.01	-2.646710782
1.0E-10	-6.880294546
0.0	-inf

(Fig. 12.03)

As long as the lower limit is >0, the integral is convergent. We then can compute the firing time from the location of peak pressure, Xp, to bullet exit, L. For our WWI Mauser example, the time would be.

$$\text{Time} = \frac{1}{3386.06} * \int_{0.06923}^{2.4417} wzl(0.1749/u)\, du = 1.02E\text{-}3 \text{ sec} = 1020 \text{ microseconds}$$

Or just over a millisecond.

FIRING TIME FROM ZERO TO Xp

The integral (12.13) was shown to be divergent at x=0. With this, how do we resolve the firing time from x=0 to x=Xp? There are two options open: (1) Scrap this entire model and find something better, or (2) Find a function, f(x) for x in [0,Xp] that has the following properties:

(1) f(x) "closely" matches (12.04) for x in (0,Xp)

(2) $f(0) = 0$
(3) $f(X_p) = v(X_p)$
(4) The time integral is convergent. That is,

$$T1 = \int_0^{Xp} \frac{du}{f(u)} < \infty \qquad (12.14)$$

Let us assume that for the interval [0,Xp] the bullet experiences linear acceleration with respect to time. We have,

$$\text{Acceleration} = h * t \quad \text{where h is rate of acceleration} \qquad (12.15)$$

Integration with respect to time gives velocity.

$$v = 0.5 * h * t^2 \qquad (12.16)$$

Integration of (12.16) with respect to time gives distance covered.

$$x = 1/6 * h * t^3 \qquad (12.17)$$

Setting x = Xp and v = Vp (velocity at Xp) and performing the algebra leads to:

$$h = \frac{2/9 * Vp^3}{Xp^2}, \qquad Tp = \left| \frac{6 * Xp}{h} \right| ^{(1/3)} \qquad (12.18)$$

Now to test for convergence. Solving (12.16) and (12.17) to generate a function v=f(x), we get:

$$v1(x) = (4.5 * h * x^2)^{(1/3)} \qquad (12.19)$$

Letting $J = (4.5*h)^{(1/3)}$ we have the integral

$$Tp = \frac{1}{J} * \int_0^{Xp} \frac{du}{u^{(2/3)}} = \frac{3}{J} * Xp^{(1/3)} \qquad (12.20)$$

which is clearly convergent.

How close does (12.19) match (12.04) for x in [0,Xp]? Let us calculate from the Mauser example:

Xp = 0.06923
Vp = 823.2 ft/sec
h = 2.5864731E+10
Tp = 2.523E-4 sec = 252 microseconds.
J = 4882.476

Finding the RMS of the difference between v1(x) and v(x) for x in [0,Xp] leads to the integral,

```
        |----------------------------------------------------
    \   |          /0.06923
     \  |    1     | |    3386                       |
  RMS = \| ------- * | | ------------- - 4882.476*x^(2/3) |^2 du   (12.21)
```

```
        | 0.06923   | | wzl(0.1749/u)                   |
        |           /0
```

The value of RMS = 12.351 ft/sec.

The average velocity of v(x) for x in [0,Xp] is,

```
            /0.06923
   3386.06   |      du
   ------- * | ------------- = 503.125 ft/sec                  (12.22)
   0.06923   | wzl(0.1749/u)
            /0
```

So the average error would be 12.351/503.125 = 2.45%

TOTAL FIRING TIMES FOR DIFFERENT POWDERS

For the gun example in Fig. 12.02 we will compute the different times to compare "fast" vs. "slow" powders. Tp is time to peak pressure, T24 is time from x=0 to 2 ft (24-in barrel), T29 is time from x=0 to x=2.4417; Vp is velocity at peak pressure. All times are given in microseconds.

	Tp	Vp	T24	T29
Fast powder	223	807	1084	1237
Actual powder in M1898	252	823	1120	1273
Slow powder	285	842	1159	1311

(Fig. 12.04)

INCREASED PERFORMANCE BY DELAYING PEAK PRESSURE

Assuming that the model presented is "correct" i.e. bullet acceleration is described by a Wexzalic function, is there a way to theoretically increase muzzle velocity? If so, how much?

Using (12.07) and (12.10) one can find the location of x such that the velocity for the given pressure is maximum. This is done by solving,

$$\frac{dp}{dx} = 0 \qquad (12.23)$$

This gets most "messy" if done by hand so we use a symbolic algebra system such as MAPLE to obtain the most interesting answer of,

$$x = k2 * L,\ k2 = \frac{1}{2} * \frac{e*sqrt(2)}{e^{sqrt(2)}} = 0.467298447 \qquad (12.24)$$

So for a rifle with a 29.3 inch barrel, the peak pressure location would be 13.69 inches down the barrel. If such a gun were built, it would have the barrel bulge (thickest part to contain the pressure) just forward of the back sight.

The following table demontrates that an Xp less than this optimal distance indicates that the powder has burned out "too quickly" while Xp values greater than the optimal indicate that the powder "ran out of barrel too soon". A WWI army rifle with a 154 grain bullet could (in theory) have a muzzle velocity of over 5000 ft/sec while keeping peak pressure under 50000 lb/in^2. (The varminters {12.02} would love it!) If "Wexzalic" gunpowder, having properties just described, could be made, it would alter the dynamics of firearms. The effect could be as dramatic as the change from blackpowder to smokeless (Rauchfrei) back in the 1880-1900.

German M1898 rifle.
Pp=47809 lb/in^2, L=29.3 in, K=154 grains, Calibre=0.323

Xp	Velocity	Firing time	
0.010	1257.25636	0.002191629457	
0.020	1738.90258	0.001714199696	
0.030	2084.78152	0.001520851323	
0.040	2358.50629	0.001414861865	
0.050	2585.44671	0.001348233662	
0.060	2778.93945	0.001302937886	
0.070	2947.07279	0.001270564672	Near actual data for M1898
0.080	3095.20687	0.001246628680	
0.090	3227.11569	0.001228507222	
0.100	3345.57606	0.001214559877	
0.200	4098.88698	0.001170103527	Minimum firing time
0.300	4476.64647	0.001176912658	
0.400	4696.29477	0.001194278604	
0.500	4833.23338	0.001214297501	
0.600	4921.54938	0.001234654588	
0.700	4979.06591	0.001254554258	
0.800	5016.03038	0.001273718267	
0.900	5038.74694	0.001292067833	
1.000	5051.29642	0.001309605844	
1.100	5056.42763	0.001326368626	
1.110	5056.61180	0.001328004098	
1.120	5056.74290	0.001329632354	
1.130	5056.82244	0.001331253446	
1.140	5056.85187	0.001332867427	Theoretic peak velocity.
1.150	5056.83260	0.001334474349	
1.160	5056.76601	0.001336074264	
1.170	5056.65341	0.001337667224	
1.180	5056.49607	0.001339253280	
1.190	5056.29525	0.001340832486	
1.200	5056.05215	0.001342404892	
1.300	5051.53457	0.001357766257	
1.400	5043.87044	0.001372502977	
1.500	5033.79946	0.001386662126	
1.900	4978.84058	0.001438330812	

Xp in feet. V in ft/sec. Time in seconds

(Fig. 12.05)

Careful examination of figure 12.05 shows that minimum firing time and peak velocity are not the same. For precision shooting, minimizing the firing time is important. The less time the entire firing sequence takes, the less chance there is of missing the target due to flinching or other unwanted movement during firing.

CONCLUSION

This chapter showed that by using data for barrel length vs. velocity, it was possible to construct a mathematical model that approximated actual practice. This model predicts that given a super slow powder that reaches peak pressure far later than actual powders, the muzzle velocity would be in far excess of today's velocities.

This model makes no claim to being the best representation of firearm behavior. A better formula that matches barrel/velocity data (as given in Fig. 12.01) that avoids the divergent integral problem and at the same time returns the correct peak pressure value/location is desired. It would be instructive for interested readers to research this topic in more detail.

12.01:

In WWI the Germans claimed a muzzle velocity of 2935 ft/sec out of a 29.3-in M1898 Mauser with a 154 grain bullet [12.05]. By WWII they converted to the M1898 with a 23.6-in barrel (K-98). This weapon produced a muzzle velocity of 2850 ft/sec with the same load. The forgoing produces the following values for our model: a = 3386.06, b = 0.1749, Xp = 0.06923 with a peak pressure of 47809 lb/in^2. The shorter firearm cost less to produce and was easier for the average solder to use. That is why the conversion was made.

12.02:

"Varminters" are shooters who like to shoot at small animals with small calibre (<0.277 in) rifles. These rifles shoot small (~90 grains) bullets at very high speeds (~3700 ft/sec) resulting in flat trajectories. The rifles have scopes on them to aid in the 300+ yards (900+ feet) shooting. Gophers, Prairie Dogs, small coyotes, etc are the (moving) targets of choice. This sport is very popular in the wide open spaces of the western U.S.A.

References for Chapter #12

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Chapter 13

The Application of the Wexzal in Automotive Testing

INTRODUCTION

It is 2 AM on Main Street in Anytown USA. The stillness of the night is shattered by the roar of engines and the squeal of tires as a 3000-lb beast begins its 1/4-mile trek in search of high speed and under 12-second runtimes.

The 1/4-mile run is the most popular way that motor vehicles (automobiles and motorcycles) are "benchmarked". The vehicle is positioned at the end of a straight road that extends for at least 1/4-mile (1320 ft). It then is accelerated from rest to maximum speed in as short a time as possible. Both the speed and time are recorded. This test measures both the power (acceleration) and (near) maximum speed of the vehicle. Many Automotive related publications such as "Road & Track", "Car & Driver", "Motorcyclist" and "Consumer's Report" report on this and other facts about motor vehicles. The 1/4-mile numbers have evolved from a test result only of interest to "Hot-rodders" to something of mainstream interest. In Europe, they use the 400 metre test in the same way.

Another figure reported is the acceleration time from a stop to 60 MPH. This number is useful for accelerating from an on-ramp onto the freeway. It is important that a vehicle be able to be at the same speed as the traffic already on the freeway as the vehicle merges with the traffic. In newer vehicles (mostly motorcycles), 60 MPH can be reached while still in first gear. The 0-60 MPH time figure is the most popular topic (behind horsepower, repairing and cost) amoung people discussing motor vehicles. Not only American ads for cars brag of rapid acceleration; most German ads have the phase "Von 0 auf Tempo 100 in x.xx Sekunde" [From 0 to 100 kilometre/hr in x.xx sec].

A vehicle's acceleration is determined by many factors. Amoung these are:

(1) Horsepower of the engine.
(2) Torque of the engine.
(3) Gearing of transmission.
(4) Weight of vehicle.

Most car publications present a graph showing the rotational speed of the engine in Rev/min (RPM) on the X-axis and the horsepower (HP) on the Y-axis. A second graph with the same RPM on the X-axis and the torque in ft-lb on the Y-axis is given. An internal combustion engine has a point where peak HP is generated. After this point the HP decreases. The torque curve has the same type of shape. The peak HP of an engine is the main factor in determining the top speed of the vehicle. The peak torque determines the peak acceleration of the vehicle.

The important thing to note is the location (in RPM) of both peak HP and torque. For a given HP, the earlier the peak torque occures on the RPM scale, the "peppier" the vehicle "feels". This "peppy" behavior comes at the expense of high speed accelerating ability. Vehicles with "late" peak torque values feel "sluggish" driving from stop-light to stop-light in a city. On a freeway, these vehicles can pass slower moving vehicles with little difficulty.

In Indiana [13.01] there was a race between a Mercedes and a Pontiac. In the best "Cannonball" tradition, the race was at night and involved travel through small towns. The Pontiac could out accelerate the Mercedes in the "Stoplight derby" (stoplight to stoplight driving) that comprised the first part of the race. Once they got on the freeway, the Mercedes was able to cruise at 130 MPH with little discomfort. The Pontiac had all it could do to maintain 110 MPH. The Pontiac's engine and gearing were designed for rapid acceleration in city driving; the Mercedes was designed with the Autobahn in mind where traffic averages over 90 MPH. The detailed analysis of transmission selection/engine design is beyond the scope of this book. {13.01}

TIME VS. VELOCITY

Because of the nature of the HP/torque curve of an internal combustion engine, the acceleration of a vehicle would not be linear. The speed would increase quickly at the start of the acceleration run and then level-off to a maximum speed. This maximum speed is called the "asymptotic velocity" of the vehicle.

Assuming that the velocity is zero at time zero and the vehicle has an asymptotic velocity, a Wexzalic function in the form of,

$$v = \frac{a}{\mathrm{wzl}(b/t)} \qquad (13.01)$$

might be a possibility. It was observed that this appears to be the case as v(0)=0, and v(inf)=a. When plotted, equation (13.01) looks very much like the "time vs. speed" curve as given in auto journals when they review a new car. The remainder of this chapter discusses the mathematical outcome of this assumption.

Like other applications, we will use standard engineering units:

a = Theoretic asymptotic velocity of vehicle in ft/sec
b = "Charging rate" in seconds
L = Length of acceleration run in ft (1320 ft = 1/4 mile)
x = Position of vehicle in ft
v = Velocity of vehicle in ft/sec
m = Logarithm conversion factor = log(e) ~ 0.43429...
t = Time in seconds
k = The dimensionless term: m*[L/(v*t)-1]
T60 = Time to 60 MPH (88 ft/sec) in seconds
X60 = Distance to 60 MPH in feet

"Road & track" and others publish the time it takes a vehicle to accelerate from 0-30 MPH, 0-40 MPH, etc. From this we can make a table such as Figure 13.01 [13.02]:

Time (sec)	Velocity (MPH)
2.8	30
4.6	40
6.3	50
8.8	60
11.3	70
14.7	80
19.0	90

(Fig. 13.01)

We wish to obtain values for the two coefficients, a,b, in (13.01) based on the data in figure 13.01. Any non-linear bi-parametric curve-fit program could be used. We used a FORTRAN program called SKRFIT (originally used to fit barrel lengths vs. muzzle velocity. See chapter &&) to obtain the coefficients to (13.01) for figure 13.01.

$$v\_in\_MPH = \frac{225.158637}{wzl(18.7498/t)} \quad , \text{RMS} = 0.52357 \text{ MPH} \qquad (13.02)$$

Converting to standard units we obtain,

$$v = \frac{330.232668}{wzl(18.7498/t)} \quad , \text{RMS} = 0.7679 \qquad (13.03)$$

Note that the Root Mean Square error is just a little over 1/2 MPH.

DISTANCE AS A FUNCTION OF TIME

From (13.01) we can calculate the distance a vehicle covers as a function of time. From physics,

$$x = \int v(t)\, dt \qquad (13.04)$$

The integral of 1/wzl(1/u) can be expressed in closed form. (See chapter && for how we obtained this result).

$$\int \frac{du}{wzl(1/u)} = \frac{u}{wzl(1/u)} + \frac{1}{m} * ei\{-2*\ln[wzl(1/u)]\} + c \qquad (13.05)$$

From this we can obtain a closed form formula for the distance a vehicle covers when given the time. Let us define,

$$S(t) = \int_0^t \frac{du}{wzl(1/u)} = \frac{t}{wzl(1/t)} + \frac{1}{m} * ei\{-2*\ln[wzl(1/t)]\} \qquad (13.06)$$

S(t) is known as the "distance function." The distance would then be

$$x = a*b*S(t/b) \qquad (13.07)$$

The acceleration equation can be calculated via,

$$\frac{d}{du} \frac{1}{wzl(1/u)} = \frac{[1/(u*wzl(1/u))]^2}{m + \frac{1/wzl(1/u)}{u}} = D(u) \qquad (13.08)$$

The acceleration of the vehicle would be,

$$\text{Acceleration\_in\_ft/sec}^2 = a/b*D(t/b) \qquad (13.09)$$

Equations (13.07) and (13.08) are written this way so a standard table containing the values: t, 1/wzl(1/t), S(t), D(t) can be used. This is for the case where no computing equipment is available.

CALCULATING 0-60 MPH TIME FROM 1/4-MILE RUN-TIME

Many "Hotrod" publications and other journals devoted to improving the performance of standard street cars and motorcycles often report only the 1/4-mile time and speed. The 0-60 MPH time is omitted as this information is more useful to commuters and other non-racing drivers.

It is possible to calculate the 0-60 MPH time and distance covered when given the 1/4-mile speed and time. Using (13.01) and (13.07) we need to solve for the constants 'a' and 'b'. Rewriting (13.07) so we can make substitution for 'a' we have,

$$a = v * wzl(b/t) \qquad (13.10)$$

where v is velocity at the 1/4-mile point and t is the time to the 1/4-mile point.

Writing out (13.07) gives,

$$L = a*b*\{\frac{t/b}{wzl(b/t)} + \frac{1}{m} * ei[-2*\ln(wzl(b/t))]\} \qquad (13.11)$$

Substituting the right hand side of (13.10) for 'a', distributing and moving all known qualitites to the left hand side gives,

$$m * (\frac{L}{v} - t) = b * wzl(b/t) * ei[-2*\ln(wzl(b/t))] \qquad (13.14)$$

Try to simplify by removing the 'b' multiplier. Let

$$z = \frac{b}{t} \qquad (13.15)$$

Substituting (13.15) into (13.14) and dividing by 't' gives,

$$k = m * (\frac{L}{v*t} - 1) = z * wzl(z) * ei\{-2*\ln[wzl(z)]\} \qquad (13.16)$$

All of the known values are on the left hand side of (13.16). We then solve for 'z' via any standard numerical method such as Newton-Raphson or Halley's method.

To obtain a close initial value for 'z', one needs to know what the right hand side of (13.16) looks like.

Let us define the "car equation",

$$car(u) = u * wzl(u) * ei\{-2*\ln[wzl(u)]\} \qquad (13.17)$$

Numerical inspection shows that,

$$\lim_{u \to 0+} car(u) = 0 \qquad (13.18)$$

For u going to infinity we have,

$$-ei(-u) \sim \frac{1}{u*exp(u)} \qquad (13.19)$$

From this, we obtain,

$$ei\{-2*ln(wzl(u))\} \sim \frac{-1}{2*ln[wzl(u)]*wzl(u)^2} \qquad (13.20)$$

Asymptote (13.20) reduces to,

$$\frac{-1}{2*wzl(u)*u/m} \qquad (13.21)$$

So we can compute the limit,

$$\lim_{u \to inf} car(u) = \frac{-u*wzl(u)}{2*wzl(u)*u/m} = -\frac{m}{2} = -0.217147241... \qquad (13.22)$$

From (13.17) thru (13.22) one can get a notion as to the behavior of car(u).
In the appendix is a table for car(u) along with S(u), D(u), V(u).

Once the value of 'z' has been computed, one then obtains the values for 'b' and 'a' via,

$$b = z * t \qquad (13.23)$$

$$a = v * wzl(b/t) = v * wzl(z) \qquad (13.24)$$

To get the T60 value one solves (60 MPH = 88 ft/sec),

$$88 = \frac{a}{wzl(b/T60)} \qquad (13.25)$$

$$\frac{a}{88.} = wzl(b/T60) \qquad (13.26)$$

Inverting the Wexzal term leads to,

$$\frac{b}{T60} = \frac{a}{88.} * log(\frac{a}{88.}) \qquad (13.27)$$

Reciprocating and multiplying by 'b' gives the final answer.

$$T60 = \frac{b}{a \qquad a} \qquad (13.28)$$

```
--- * log(---)
88.       88.
```

EXAMPLE OF CALCULATION OF T60 FROM 1/4-MILE FIGURES

The 1990 Nissan 300-ZX twin turbo [13.03] has the following 1/4-mile figures:

```
v = 101 MPH = 148.13333 ft/sec
t = 14.1 seconds
```

Using (13.16) we obtain for the left hand side,

```
                 1320.00
0.4342944819 * (------- - 1) = -0.1598298841 = k
                 2088.68
```

Solving for 'z' in (13.16) returns the value,

```
z = 1.391953175
wzl(z) = 2.956608679
```

Using (13.23) and (13.24) we obtain the values for 'b' and 'a'.

```
b = z * t = 19.62653977
a = v * wzl(z) = 437.9722989
```

Using (13.28) gives us the T60 value.

```
T60 = 5.65808464 seconds.
```

The actual T60 time for this vehicle is 5.60 seconds. This amounts to an error of 1.04%. Note that different journals will report slightly different values for the vehicle under consideration. This is caused by variance in test-driver technique, location/conditions of race-track where test is performed and variation in the vehicles themselves.

The computed distance covered is:

```
X60 = a * b * S(T60/b) = 438 * 19.6 * S(0.2882874264)

8595.880742 * 0.0348732485 = 299.7662852 ~ 300 feet.
```

The Nissan 300 ZX is a high performance sports car {13.02}. This vehicle requires about 300 feet to accelerate from a dead stop to 60 MPH.

This is not the quickest. Motorcycles can (in general) out-run automobiles due to their better power to weight ratio. One of the fastest motorcycles is the Kawasaki ZX-11 {13.03}. The ZX-11 is a 1100 cc motorcycle that weighs about 600 lb and has a 100+ HP engine.
Its performance figures are as follows [13.04]:

```
Quarter mile speed & time: 131.82 MPH in 10.52 seconds.
Actual T60 = 2.65 seconds.
Computed T60 = 2.53953 seconds
Error in T60 = 4.17%
Computed distance covered in computed T60 time = X60 = 133.7 feet
```

FIELD CALCULATION OF 0-60 MPH WITHOUT USE OF COMPUTER

Calculation of the 0-60 MPH time from 1/4-mile data via equations (13.16) thru (13.28) is best left to a programmable calculator or small computer due to the complexity of the equations. With the use of a special graph and a simple 4-function calculator, it is possible to calculate the same quantities without the use of complex formulae.

An "L"-graph is a Sklaric graph with three curves plotted on it:

(1) y = wzl(x)
(2) y = 100 * S(0.10*x)
(3) y = -10 * u * wzl(u) * ei{-2*ln[wzl(u)]} where u = x/10

As it can be surmised, the first equation is for computing velocity; the second is for distance and the third is a graphic representation of equation (13.17) but with its sign reversed. A Sklar graph shows best detail for values <10 as 10 is 82.67% of the total Sklaric axis. Thru experimentation it was found that the powers of 10 chosen for the second and third equations gave the greatest ease of reading values for the range most likely encountered in working actual 1/4-mile problems. If a class of vehicle under study (e.g. drag racers or rocket cars) require different powers of 10 for ease of reading on the "L"-graph, then just replot the three equations as required.

The three equations when plotted on the same Sklar graph produce a script letter "L". The first part of the "L", running from (0,1) to (inf,inf), is the graph of the first equation listed [y=wzl(x)]. The "stem" of the "L" running from (0,0) to (inf,inf) is the distance equation. The last part running from (0,0) to (inf,m/2) is (13.17) with its sign reversed.

It is difficult to obtain more than 2-3 decimal places when reading anytype of analogue display (graphs, slide rules, gauges, etc). Because of this, the results from the "L" graph will be approximate at best. There is one advantage of graphs over straight numeric calculation: The graph of a function is also the graph of the inverse of that function. Instead of reading the 'x' value and then reading the y(x) from graph, one just reads the 'y' value and looks on the graph for x(y). For non-linear functions that are difficult (computationally intensive) to invert, this offers a solution provided that 2-3 decimals of precision is all that is needed.

The following describes how to solve a 0-60 MPH problem with the use of a 4-function calculator (8 decimal place display) and an "L" graph. An example calculation will be given at the same time to demonstrate the procedure. The phrases "first", "second", "third" equations refer to the three equations described before that compose the "L" graph.

Given: A "generic" car with the quarter-mile speed/time of:

v = 83 MPH = 121.73333 ft/sec
t = 17.0 seconds.

Step #1:

Calculate: $$k = m * \left(1 - \frac{L}{t*v}\right) = 0.434 * \left(1 - \frac{1320}{121.73*17}\right) = 0.157282$$

'm' has the exact value of 0.4342944819 but the value 0.434 can be used.

Step #2:

Look along the 'y' axis of the "L" graph at 10*k and scan across
until the 'x' value of the third equation is hit. Looking at
y = 1.57 and scanning the third equation results in 'x' value
being 12. Because of the "10*" and "0.1*" in the third equation,
the 'x' value is really 10*x. Take the answer from the graph
and divide by 10. The solution of the third equation is
then z=1.2

Step #3:

Calculate 'b' via b = z * t.
b = 1.2 * 17 = 20.4

Step #4:

We are now going to compute the 'a' value. For this we need
the Wexzal of 'z'.
Using the 'z' value, scan on the 'x' axis and read the 'y' axis
value for the first equation. There are no multiply/divide factors
due to the first equation being un-"biased" by a power of 10.
wzl(1.2) ~ 2.7
From this, we get,
a = v * wzl(z) = 121.73 * 2.7 = 328.67

Step #5:

At this point, we have both the 'a' and 'b' values. Now to compute
the T60 value via eqn (13.28). This formula involves logarithms
which our calculator doesn't have. But note the denominator
is in the form of 'x * log(x)' which is the inverse of the
Wexzal function. Calculate a/88 and look on the 'y' axis for this
value. Scan along 'y' axis until the first function is hit.
Read the 'x' value.
a/88 = 328.67 / 88 ~ 3.73
The Inverse Wexzal of 3.73 (from graph) is 2.14
Using (13.28) calculate T60.
T60 = 20.4 / 2.14 = 9.53 Seconds

Step #6:

To find the distance to 60 MPH requires using (13.07).
Calculate T60/b and multiply by 10. Look for this value on the
'x' axis and look for the 'y' value of the second equation.
Take this result and divide by 100. Now multiply by (a*b).
T60/b = 9.53 / 20.4 = 0.467
Multiply by 10 gives 4.67. Use 4.7 for Sklar graph as detail
decays for increasing 'x' values.
Reading second equation gives y = 7.7
Division by 100 gives 0.077.
Final answer is:
X60 = a*b*S(T60/b) = 328.67 * 20.4 * 0.077 = 516 feet.

Because of human error when reading graphs, error creeps in. The
forgoing problem when run on a programmable calculator give the
following results:

k = 0.1572817011
z = 1.204076394
b = 20.4692987

a = 334.1967219
T60 = 9.300694644
X60 = 503.8147985

Assuming that the calculator result is "exact" and "correct" the error value for T60 and X60 is 2.47% and 2.42%

CONCLUSION

This chapter examined the mathematical outcome of assuming that a motor vehicle's acceleration can be modelled using a Wexzalic function. This resulted in a method to compute, to within a few percent error, the time and distance required to accelerate from a dead stop to 60 MPH when given the quarter mile speed and time.

There are a few defects with this model:

(1) D(0) = inf. This means that at the start of the acceleration run, the vehicle undergoes infinite acceleration. This is of course not correct. The integral of the acceleration function D(u),

$$\int_0^t D(u)\,du = \frac{1}{wzl(1/t)}$$

is convergent however.

(2) The theoretic asymptotic velocity is too high. In the sports car example above, the theoretic asymptotic velocity was 438+ ft/sec ~ 300 MPH. For that vehicle, the actual asymptotic velocity is on the order of 200 MPH (293.3 ft/sec). This means that this model cannot be used to extrapolate vehicle behavior past the 1/4 mile mark.

(3) There is no physical justification for the use of Wexzals to model vehicle behavior. The good agreement for distances under 1/4-mile stems from the notion that the Wexzalic function used somehow ties together the effects of engine HP/torque, tire rolling resistance, aerodynamic drag on the vehicle and other secondary non-linear effects into one "tidy" simple equation. The net result is all that this model attempts to simulate.

Today, with the advent of super fast computers and calculators that have the power of 1960's era minicomputers, the study of non-linear effects and models should be expanded. Wexzals might be just one way to describe non-linear behavior of physical systems.

13.01:

Engine/transmission mating is not a simple thing. Many "backyard" mechanics attempt to "beef-up" the performance of their street cars by just simply installing a bigger engine that will fit. This sometimes result in broken transmissions due to the increased torque of the bigger engine. These innocent looking cars, known as "sleepers", are noted for being able to accelerate far quicker than expected; sometimes to the tune of squealing tires and belching flames out the exhaust pipe as the engines are (sometimes) feed too much gasoline. Beware of that beat-up 1972 Toyota Corolla that shakes when idling at a stop light...

13.02:

In the U.S.A. the government passed a law requiring that all high performance cars, e.g. Corvette, 300-ZX, etc, be governed (by mechanical or computer) to a top speed of 155 MPH (227 ft/sec). Considering that the maximum speed allowed is 65 MPH (95 ft/sec) and that speeds over 85-90 MPH (in most states) result in *large* fines and jail time, it seems pointless to limit a motor vehicle that can travel faster than a small airplane to such a "half-hearted figure". If a limit has to be made, why not something like 90 MPH?

Clever after-market car-parts suppliers have a way to "walk-around" the limit by replacing or re-programming the onboard coomputer. When "uncorked", these vehicles can reach speeds of over 200 MPH.

13.03:

High performance motorcycles like the ZX-11 (which is the most powerful of the "Ninja" series of sportbikes) have become the bane of insurance companies and law makers. This is caused by inexperienced riders purchasing these vehicles and then getting into fatal accidents. Many car drivers are fearful of motorcyclists because of difficulty seeing the motorcycle in traffic and the motorcyclist (sometimes) taking advantage of his better accelerating ability in changing lanes.

Motorcycles are very interesting from the standpoint of the math model under discussion. A middle-sized (in the U.S.A. that means a 750cc) motorcycle can out accelerate all but the fastest cars. Yet most motorcycles average 40 miles to the gallon which is better than all but the most efficient cars. Motorcycles are good; do not pick on them.

References for Chapter #13

----

(1) Mary Alice Grossman - Aerospace engineer for U.S.A.F.
Private communication.

(2) Nissan 240 review
Road & Track, September 1988

(3) Nissan 300ZX review
Road & Track, September 1990

(4) ZX-11 review
Motorcyclist, March 1990

Chapter 14

Table of Inequalities and Identities

INTRODUCTION

The following chapters contain tables and charts of formulae and numbers discussed in the body of this work. They represent our current collection of all known (to the authors) facts about Wexzals and related functions.

To save space, we will use a computer-programming-like notation to represent integrals and other formulae. So,

```
                   /b
                   |
ing[a,b,f(x) dx] = | f(x) dx
                   |
                   /a


               /
               |
ing[f(x) dx] = | f(x) dx = g(x) + c
               |
               /
```

The following are basic constants used,

```
e = exp(1) = 2.718...

m = log(e) = 0.434...

i = sqrt(-1)
```

The following functions are,

```
y=x^x = cxt(x)                      Coupled Exponent

x=y^y,  y=crt(x)                    Coupled Root

wzl(x) = crt(10^x)                  Wexzal Function

x=y^y^y, y=trp(x)                   Tripled Root


ei(x) = ing[-inf,x,e^u/u du]        Exponential Integral

pvr(x) = ing[0,x,u/wzl(e^u) du]     "Pulver" (Powder) Integral

gi(x) = ing[1,x,crt(u) du]          Coupled Exponent Integral

wi(x) = ing[0,x,10^u*wzl(u) du]

ri(x) = 1/m*ing[0,x,10^u/wzl(u) du]
```

```
ji(x) = 1/m*ing[0,x,wzl(u)/10^u du]

P(x) = ing[dx/(x*wzl(x))] = 1/wzl(x)-ei{-ln[wzl(x)]}

B(x) = ing[wzl(x)/x dx] = wzl(x)+ei{ln[wzl(x)]}

S(x) = ing[dx/wzl(1/x)] = x/wzl(1/x)+1/m*ei{-2*ln[wzl(1/x)]}
```

For the following, x,y,v are real numbers over the entire real axis unless otherwise restricted.

```
(01) log[wzl(x)] = x/wzl(x)                    Logarithmic identity

(02) wzl(x*10^x) = 10^x

(03) log{cxt[wzl(x)^2]} = 2*x*wzl(x)

(04) x^[x/log(x)] = 10^x

(05) sqrt{cxt[wzl(x)^2]} = 10^[x*wzl(x)]

(06) cxt[wzl(x)] = 10^x

(07) cxt(x*y) = [y^(x-1)*cxt(x)]^y*cxt(y)

(08) cxt(x^v) = cxt(x)^[v*x^(v-1)]

(09) cxt[v*crt(x)]/x^v = cxt(v)^crt(x), v>0

(10) log(x)/x = {1+log[log(wzl{x})]/log[wzl(x)]}/wzl(x)

(11) 10^{1+log[log(wzl{x})]/log[wzl(x)]} = x^[wzl(x)/x]

(12) log[trp(x)] = log(x)/cxt[trp(x)]

(13) wzl(x)>x for x in [0,10)
     wzl(x)=x for x=10
     wzl(x)<x for x>10

(14) wzl(x) + wzl(y) > wzl(x+y) for x>=0, y>=0

(15) v*wzl(x) > wzl(v*x) for v>1
     v*wzl(x) = wzl(v*x) for v=1
     v*wzl(x) < wzl(v*x) for 0<=v<1

(16) x^x^x > x^x > crt(x^x^x) > crt(x)^x > x > cxt[trp(x)] > trp(x^x)
     trp(x^x) > crt(x) > trp(x) for x>1         The ordering property
```

Chapter 15

Solutions of Equations in Closed Form

----

(01) y=x^x,   x=crt(y)

(02) y=x*log(x), x=wzl(y)

(03) y=x*10^x, x=y/wzl(y)

(04) y=x^2*log(x), x=sqrt[wzl(2*y)]

(05) y=x*log(x)^2, x=wzl[0.5*sqrt(y)]^2

(06) y=x+log(x), x=log[wzl(10^y)]

(07) y=x+wzl(x), x=log{cxt(0.1*wzl(10*y)]}

(08) y=x*wzl(x), x=log{cxt[sqrt(wzl{2*y})]}

(09) y=x^(1/x), x=1/crt(1/y)

(10) y=x^[x*log(x)], x=wzl{0.5*sqrt[log(y)]}^2

(11) y=x^x^2, x=sqrt{wzl[2*log(y)]}

(12) y=x*10^sqrt(x), x=log{wzl[0.5*sqrt(y)]^2}^2

(13) x-y^(-x)=0, x=1/crt(y)

(14) y=crt(x)*log(x), x=cxt{sqrt[wzl(y)]}

(15) y=sqrt(x)*log(x), x=wzl(0.5*y)^2

(16) 1=crt(x)*log(x)/log(y), x=cxt{sqrt[crt(y^2)]}

(17) y=x+10^x, x=log{log[wzl(10^y)]}

(18) y=x+x*log(x), x=0.1*wzl(10^y)

(19) y=x^[x*log(x)^2], x=wzl[1/3*log(y)^(1/3)]^3

(20) y=x^[x^2*log(x)], x=wzl{sqrt[log(y)]}

(21) wzl(x)=y*x, x=10^(1/y)/y

(22) wzl(x)=y^x, x=log{cxt[1/log(y)]}

(23) wzl(x)=x^y, x=log{cxt[wzl(1-1/y)^{1/(1-1/y)}]}

(24) y=x^2*wzl(x), x=log{cxt[wzl(3/2*sqrt[y])^(2/3)]}

(25) y=x*wzl(x)^2, x=log{cxt[wzl(3*y)^(1/3)]}

(26) y=x^2/wzl(x)=x*log[wzl(x)], x=log{cxt[wzl(0.5*sqrt{y})^2]}

(27) y=wzl(x)^x, x=log{cxt[wzl(0.5*sqrt[log{y}])^2]}

(28) y=crt(x)^log(x), x=cxt[wzl(0.5*sqrt[log{y}])^2]

(29) y=x^x*10^x, x=0.1*wzl[10*log(y)]

(30) y=sqrt(x)^x, x=wzl[2*log(y)]

(31) y=(2*x)^x, x=0.5*wzl[2*log(y)]

(32) y=x^sqrt(x), x=crt[sqrt(y)]^2

(33) y=x*10^x+log(x), x=log[trp(10^10^y)]

(34) y=x^x*e^x, x=1/e*wzl[e*log(y)]

(35) y=x*e^crt(x), x=cxt{1/e*wzl[e*log(y)]}

(36) y=10^x*e^wzl(x), x=log{cxt[1/e*wzl(e*log{y})]}

(37) y=1/wzl(1/x)=y*x, x=1/(y*10^y)

(38) y=1/wzl(1/x)=x^y, x=1/{log[cxt(wzl{1-1/y}^[1/(1-1/y)])}

(39) y=x*10^x/wzl(x), x=log{cxt[trp(10^y)]}

(40) y=wzl(x)^(10^x), x=log{cxt[trp(y)]}

(41) y=x*log(x)/crt(x), x=cxt[trp(10^y)]

(42) y=x*ln(x)+x, x=1/e*wzl(m*e*y)

(43) y=c^x, x=c^y, y=x @ x=1/crt(1/c)

(44) y=x*log(x)^2+x*log(x)*log[log(x)], x=wzl[wzl(y)]

(45) y=x-wzl(x), x=y+10*wzl(0.1*y)

(46) y=x*log(x)-x, x=10*wzl(0.1*y)

(47) y=x^x/10^x, x=10*wzl[0.1*log(y)]

(48) y=log(x)/crt(x), x=cxt(10^y)

(49) (10^x)^y=x^x, x=10^y

(50) y=x*wzl(1/x), x=y/10^(1/y)

(51) wzl(x)=x^x, x=trp(10)=1.923584036...

(52) log(x)=1/x, x=crt(10)=2.50618414559...

Chapter 16

Integrals Given in Closed Form

```
(01) ing[wzl(x) dx] = 0.5*x*wzl(x)+m/4*[wzl(x)^2-1]+c

(02) ing[dx/wzl(1/x)] = x/wzl(1/x)+1/m*ei{-2*ln[wzl(1/x)]}+c = S(x)+c

(03) ing{ing[0,x,du/wzl(1/u)] dx} =
        x^2/wzl(1/x)+x/m*ei{-2*ln[wzl(1/x)]}+1/m^2*{m*x/[2*wzl(1/x)^2]*
        [1-m*x*wzl(1/x)]+3/2*ei[-3/m*1/[x*wzl(1/x)]]}+c

(04) ing[wzl(x)/x dx] = wzl(x)+ei{ln[wzl(x)]}+c = B(x)+c

(05) ing[dx/(x*wzl(x))] = 1/wzl(x)-ei{-ln[wzl(x)]}+c = -P(x)+c

(06) ing[wzl(1/x) dx] = 1/m*{1/ln[wzl(1/x)]-ln[ln(wzl{x})]}+c

(07) ing[P(x) dx] = x*P(x)+x/wzl(x)*{1+x/[2*m*wzl(x)]}+c

(08) ing[P(1/x) dx] = x*P(1/x)-x/wzl(1/x)-1/m*ei{-2*ln[wzl(1/x)]}+c

(09) ing[ei{-ln[wzl(x)]} dx] = -x*P(x)+c

(10) ing[P(x*log(x)) dx] = ln(x)+ln[ln(x)]+0.577216-x*ei[-ln(x)]+c

(11) ing[P(x)/{m+x/wzl(x)} dx] = P(x)*wzl(x)+ln(x)+c

(12) ing[x*P(x) dx] = 0.5*{x^2*P(x)+x^2/wzl(x)-[m*x-m^2*(wzl(x)-1)]}+c

(13) ing[x*P(1/x) dx] =
        0.5*{x^2*P(1/x)+1/m^2*{m*x/[2*wzl(1/x)^2]*[1-m*x*wzl(1/x)]+
        3/2*ei[-3/(m*x*wzl(1/x))]]}+c

(14) ing[ei{-ln[wzl(x)]}/x^3 dx] =
        0.5/x^2*{1/wzl(x)-ei[-ln(wzl(x))]}+1.5/m^2*{m/[2*x*wzl(x)^2]*
        [1-m*wzl(x)/x]+3/2*ei[-3*x/(m*wzl(x))]}+c

(15) ing[x/wzl(1/x) dx] =
        -1/m^2*{m*x/(2*wzl(1/x)^2)*[1-m*x*wzl(1/x)]+3/2*
        ei[-3/m*1/(x*wzl(1/x))]}+c

(16) ing[dx/(x^3*wzl(x))] =
        1/m^2*{m/[2*wzl(x)^2*x]*[1-m*wzl(x)/x]+3/2*ei[-3*x/(m*wzl(x))]}+c

(17) ing[wzl(1/x)/x dx] = -{wzl(1/x)+ei[ln(wzl(1/x))]}+c

(18) ing[x*wzl(1/x) dx] =
        1/m^2*{0.5*[(m*x)^2*wzl(1/x)+m*x+ei{-ln[wzl(1/x)]}]}+c

(19) ing[crt(x) dx] = x*crt(x)-1-gi[crt(x)]+c

(20) ing[dx/(x*crt(x)^2)] = {-ln[crt(x)]-2}/crt(x)+c

(21) ing[log{cxt[wzl(x)*wzl(1/x)]} dx] =
        1/m^2*{0.5*[(m*x)^2*wzl(1/x)+m*x+ei{-ln[wzl(1/x)]}]}+wzl(x)+
        ei{ln[wzl(x)]}+c
```

```
(22) ing[log[wzl(x)*wzl(1/x)] dx] =
        x^2/wzl(x)-[m*x-m^2*(wzl(x)-1)]+1/wzl(1/x)-ei{-ln[wzl(1/x)]}+c

(23) ing[dx/(x*wzl(1/x))] = 1/wzl(1/x)-ei{-ln[wzl(1/x)]}+c

(24) ing[wzl(x)^2/x dx] = ei{2*ln[wzl(x)]}+0.5*wzl(x)^2+c

(25) ing[log(x)*wzl(x) dx] =
        A*log(x)-m*{0.5*A+m/4*{ei[2*ln(wzl(x))]}+0.5*wzl(x)^2-ln(x)}+c
        where A=ing[0,x,wzl(u) du]   See Integral #1

(26) ing[dx/wzl(x)^3] = -1/[2*wzl(x)^2]*[3*m/2+x/wzl(x)]+c

(27) ing[dx/wzl(x)^4] = -m/[3*wzl(x)^3]*[x/(m*wzl(x))+4/3]+c

(28) ing[crt(x)/x dx] = 0.5*ln(x)*crt(x)+0.25*[crt(x)^2-1]+c

(29) ing[0.5*sqrt{2*log[wzl(2*100^x)]} dx] =
        1/12*T^3+m/2*T+c  where T=sqrt{2*log[wzl(2*100^x)]}

(30) ing[dx/(x*crt(x))] = ln(x)/crt(x)*[1+ln(x)/(2*crt(x))]+c

(31) ing[dx/wzl(x)^2] = x/wzl(x)^2-{m/wzl(x)*[2+ln[wzl(x)^2]]-2*m}+c

(32) ing[dx/(x+x^2/wzl(m*x))] = ln(x)-x/wzl(m*x)+c

(33) ing[dx/(1+ln[crt(x)])] = x*crt(x)-1/m*wi[log(x)]+c

(34) ing[x*wzl(x) dx] =
        1/3*wzl(x)^3*{m*[x/wzl(x)-m/3]+(x/wzl(x))^2-2/3*m*[x/wzl(x)-m/3]}+c

(35) ing[wzl(0.5*sqrt(x))^2 dx] =
        x*wzl(0.5*sqrt(x))^2-{0.5*wzl(0.5*sqrt(x))^4*[x/wzl(0.5*sqrt(x))^2-
        m*sqrt(x)/wzl(0.5*sqrt(x))^2+m^2/2]-m^2/4}+c

(36) ing[sqrt(wzl(2*x)) dx] =
        x*sqrt[wzl(2*x)]-{wzl(2*x)^1.5/3*[x/wzl(2*x)-m/3]+m/9}+c

(37) ing[x/wzl(x) dx] = x^2/wzl(x)-{m*x-m^2*[wzl(x)-1]}+c

(38) ing[dx/wzl(x)] = x/wzl(x)*[1+x/(2*m*wzl(x))]+c

(39) ing[wzl(x)^2 dx] = x*wzl(x)^2-{wzl(x)^3/3*[2*x/wzl(x)-2/3*m]+2/9*m}+c

(40) ing[dx/sqrt(wzl(x))] = 2*m*sqrt[wzl(x)]*[ln(wzl(x))-1]+c

(41) ing[wzl(x)/x^2 dx] = 1/m*{ln(ln(wzl(x)))-1/ln[wzl(x)]}+c

(42) ing[wzl(x)/x^3 dx] = 1/m^2*{-0.5*[m^2*wzl(x)/m^2+m/x+ei{-ln[wzl(x)]}]}+c

(43) ing[log(wzl(10^x)) dx] = log[wzl(10^x)]*{0.5*log[wzl(10^x)]+m}+c

(44) ing[dx/crt(x)] = ri[log(x)]+c

(45) ing[dx/x^x] = x/x^x+ji[x*log(x)]+c

(46) ing[(wzl(x)/x)^2 dx] = 2/m*ei{ln[wzl(x)]}-wzl(x)^2/x+c

(47) ing[B(x) dx] = 0.5*x*wzl(x)-m/4*[1+wzl(x)^2]+x*ei{ln[wzl(x)]}+c

(48) ing[ei{ln[wzl(1/x)]} dx] = x*{ei{ln[wzl(1/x)]}+wzl(1/x)}+c
```

(49) ing[-ei{-ln[wzl(1/x)]} dx] =
-x*ei{-ln[wzl(1/x)]}+1/m*{-m*x/wzl(1/x)-2*ei{-2*ln[wzl(1/x)]}}+c

(50) ing[ei{ln[wzl(x)]} dx] = x*ei{ln[wzl(x)]}-m/2*wzl(x)^2+c

(51) ing[-ei{-ln[wzl(x)]} dx] = x*P(x)+c

(52) ing[sqrt(wzl(1/x)) dx] =
1/m*{-1/[sqrt(wzl(1/x))*ln(wzl(1/x))]+0.5*ei{-0.5*ln[wzl(1/x)]}}+c

(53) ing[ei{-2*ln[wzl(1/x)]} dx] = m/wzl(x)+x*ei{-2*ln[wzl(x)]}+c

(54) ing[P(x)/x^2 dx] = 1/[x*wzl(x)]-P(x)/x+1/m*ei{-2*ln[wzl(x)]}+c

(55) ing[P(x)/x dx] = ln(x)*pvr(x)+pvr[ln(x)]+c

(56) ing[S(1/x) dx] = x*S(1/x)-P(x)+c

(57) ing[P(1/x) dx] = x*P(1/x)-S(x)+c

(58) ing[sqrt(wzl(x)) dx] = m*{[2/9+2/3*ln(wzl(x))]*e^[3/2*ln(wzl(x))]}+c

Chapter 17

Asymptotics and Limits

```
(01) wzl(x) ~ x/log(x)

(02) trp(10^x) ~ 1+crt{x/[e*log(x)]}

(03) y=x!,  x ~ e*wzl{1/e*log[y/sqrt(2*pi)]}-0.5

(04) crt(x!) ~ x*[1-1/ln(x)]

(05) d[wzl(x)]/dx = 1/[m+x/wzl(x)] ~ wzl(x)/x ~ 1/log(x)

(06) wzl(x+1) ~ wzl(x)*e^{1/wzl(x)*d[wzl(x)]/dx}

(07) wzl(2*x) ~ wzl(x)^{1+log(2)/[m+x/wzl(x)]}

(08) lim {wzl(x)^[d(wzl(x))/dx]} = 10
     x->inf

(09) wzl(1/x) ~ 1+1/(m*x)-0.5/(m*x)^2+(2/3)/(m*x)^3+...

(10) 1/m*ei{-2*ln[wzl(1/x)]} ~ 4.845549226-ln(x)/m-10.60379622/x

(11) y=(2*x)!/x!, x ~ e/4*crt{[y/sqrt(2)]^(4/e)}

(12) P(1/x) ~ -0.4112481102+ln(x)+1/(m*x)-3.976/x^2+10.85/x^3+...

(13) 1*4*27*256*...n ~ 1.2824271*n^(n^2/2+n/2+1/12)/e^(n^2/4)

(14) trp(x^x) ~ 1+crt{x/e*[1-1/crt(x)]}

(15) log[wzl(x+1)] ~ x/wzl(x)+1/[x/m+wzl(x)]

(16) crt(2*x)/crt(x) ~ 1+log(2)/[m*crt(x)+log(x)]

(17) log{wzl[x+log(x)]} ~ (m+x)/wzl(x)

(18) wzl[x+log(x)] ~ wzl(x)+1

(19) crt[x^(x+1)] ~ x+1/[1+1/ln(x)]

(20) crt[crt(x^x^x)] ~ x-1+1/[1+ln(x-1)] ~ x-1

(21) crt{wzl[f(x)]} ~ crt[f(x)]-1  such that f(x)>=1 for all x, f(inf)=inf

(22) trp[cxt(x^x)] ~ x+1/[1+1/ln(x)] ~ x+1

(23) wzl(x^2+x) ~ wzl(x^2)+x/[2*log(x)]

(24) wzl[x^x*log(x)] ~ x^x/x*[1+1/x]

(25) wzl[x*log(x)^2] ~ x*log(x)/e^{2*log[log(x)]/log(x)}

(26) log[wzl(10*x)] ~ x/wzl(x)+1/[1+m*wzl(x)/x] ~ 1+x/wzl(x)

(27) cxt(x+1)/cxt(x) ~ e/2+e*x-e/(24*x)+e/(48*x^2) ~ e*x+e/2
```

```
(28) [1/wzl(1/(2*x))]/[1/wzl(1/x)] ~ 1+1/(2*m*x)-5/[8*(m*x)^2]+...

(29) 1/wzl(1/x)^2 ~ 1-2/(m*x)+4/(m*x)^2-25/[3*(m*x)^3]+...

(30) crt(1+1/x) ~ 1+1/x-1/x^2+3/(2*x^3)+...

(31) trp(1+1/x) ~ 1+1/x-1/x^2+3/(2*x^3)+7/(6*x^4)+...

(32) 1/wzl[1/(x+1)] ~ 1/wzl(1/x)*[1+1/(m*x^2)-(m+2)/(m^2*x^3)+...]

(33) 1/wzl(1/x) ~ 1-1/(m*x)+3/[2*(m*x)^2]-8/[3*(m*x)^3]+...

(34) y(x)=log[wzl(10^x)] ~ x-log(x),  y(-x) ~ 1/[1/m+10^x]

(35) y(x)=x^x/x ~ e*cxt(x-1), y(1/x) ~ x-ln(x),
                          x1 ~ 1+crt(y/e), x2 ~ 1/{y+ln[y+ln(y)]}

(36) y(x)=x^x*x ~ 1/e*cxt(x+1), x ~ crt(e*y)-1

(37) y=wzl(x)*log(x), x ~ y/{1+log[log(y/log(y))]/log(y/log(y))}

(38) y=(x+1)^x ~ e*cxt(x), x ~ crt(y/e)

(39) y=x^2+log(x), x=0.5*sqrt{2*log[wzl(2*100^y)]} ~ sqrt(y)

(40) wzl(x)^2 ~ 2*wzl[x*wzl(x)]

(41) y=x*10^(x^2), x=sqrt{0.5*log[wzl(2*y^2)]} ~ sqrt[log(y)]

(42) y(x)=wzl(x)/x ~ 1/log(x), y(1/x) ~ 1/m+x

(43) wzl{sqrt[x*log(x)]} ~ sqrt[wzl(4*x)]

(44) wzl(v*x) ~ v*wzl(x), for v>0

(45) wzl[0.5*sqrt(x)] ~ wzl(x)/log(x)

(46) 2*log{wzl[0.5*10^(x/2)]} ~ x-2*log(x)

(47) wzl{[x*log(x)]^v} ~ x^v/v*log(x)^(v-1), for v>0

(48) wzl(x^v*10^x) ~ x^(v-1)*10^x, for v>0

(49) wzl[x^v*log(x)] ~ x^v/v, for v>0

(50) log[wzl(x^v)] ~ v*log[wzl(x)], for v>0

(51) wzl(x+y)-wzl(x)=C, y ~ C/{d[wzl(x)]/dx}

(52) lim P(x)*wzl(x)+ln(x) = 1-gamma-ln[ln(10)] = -0.4112481102
     x->0

(53) S(x) ~ x+2.542964134-ln(x)/m-7.95285/x+16.277/x^2

(54) f(x)=ing[1,e^x,wzl(u)/u du] ~ e^x*[1/(m*x)+(0.382+ln(x)/m)/x^2]

(55) f(1/x) ~ wzl(1)/x+0.600/x^2+0.15/x^3+0.027/x^4+0.0044/x^5+...

(56) B(1/x) ~ 2.41124811-ln(x)+1/(m*x)

(57) ing[0,x,u/wzl(1/u) du] ~ x^2/2-x/m-10.68221448+7.952847*ln(x)+32.55/x
```

(58) ing[0,x,S(u) du] ~ x^2-1/m*[-2.104395291+ln(x)]*x+2.729367-7.95*ln(x)

(59) 1/m*ing[1/x,inf,wzl(u)/10^u du] ~ 1.70417-1/(m*x)+4.069/x^3

(60) ing[1+1/x,inf,du/u^u] ~ 0.70416996-1/x+1/(2*x^2)-1/(8*x^4)+...

(61) 1/x*ing[1,e^x,du/(u*wzl(u))] ~ P(1)/x+1/e^x*[-m-m*({1.8-ln(x)}/x)]

(62) pvr(1/x) ~ 0.19951/x^2-0.06368652/x^3+0.0049487/x^4+...

(63) ing[x,inf,[wzl(1/u)-1]/wzl(u) du] ~ P(x)/m-S(1/x)/(2*m^2)

(64) S(1/x) ~ 1/[2*x*wzl(x)]

(65) Laplace transform of wzl(m*x) ~ 1/s+1/s^2-1/s^3+4/s^4-27/s^5+256/s^6+...

(66) P(x)*wzl(x)+ln(x) ~ 1+ln(x)

(67) ing[0,x,u*P(u) du] ~ x^2/wzl(x)

(68) y=gi(x) ~ x^x/[1+ln(x)], x ~ crt{y*[1+ln(crt(y))]}

(69) wi(x)~10^x*{m*wzl(x)-m^2/[m+x/wzl(x)]-m^4/wzl(x)*[1/(m+x/wzl(x))]^3}

(70) w(1/x) ~ 1/x+1/(m*x^2)+1/(3*m^2*x^3)+5/(24*m^3*x^4)+...

(71) ing[0,1/x,wzl(u) du] ~ 1/x+1/(2*m*x^2)+...

(72) ing[0,x,wzl(u) du] ~ 0.5*x*wzl(x)

(73) P(x) ~ 1/wzl(x)+m/x*[1-1/ln(x)]

(74) ing[x,inf,du/u^u] ~ 1/[x^x*(1+ln(x))]*{1-1/[x*(1+ln(x))]}

(75) ing[x,inf,wzl(u)/10^u du] ~ m*wzl(x)*[1+1/(x/m+wzl(x))]

Chapter 18

Special Values

```
(01) wzl(1) = 2.50618414559

(02) 1/wzl(1) = log[wzl(1)] = 0.39901297826

(03) trp(10) = 1.9235840364

(04) P(1) = 0.650886653739

(05) B(1) = 4.18021883536

(06) S(1) = 0.257713187868

(07) ing[0,1,crt(1/x) dx] = ing[1,inf,crt(x)/x^2 dx] = ji(inf) =
     1 + ing[1,inf,dx/x^x] = 1.7041699552

(08) ing[0,1,dx/crt(1/x)] = ing[0,inf,dx/(e^x*wzl(m*x))] = 0.6465032

(09) 1/m*ing[0,inf,dx/wzl(x)^v] = v/(v-1)^2  such that v>1

(10) ing[1,inf,dx/(x*crt(x)^2)] = 2

(11) B(x)=0, x=0.0758335698276

(12) ing[wzl(1/x) dx]=1/m*{1/ln[wzl(1/x)]-ln[ln(wzl(1/x))]}=0, x=0.2239497283

(13) ing[1,inf,wzl(1/x)/x^2 dx] = ing[0,1,wzl(x) dx] = 1.826464908

(14) pvr(inf) = ing[1,inf,P(x)/x dx] = 0.936276967
```

****

x	wzl(x)	crt(x)	trp(x)
1.00	2.506184146	1.000000000	1.000000000
2.00	3.597285024	1.559610469	1.476684337
3.00	4.555535705	1.825455023	1.635078475
4.00	5.438582696	2.000000000	1.722191913
5.00	6.270919556	2.129372483	1.780037839
6.00	7.065796728	2.231828624	1.822418026
7.00	7.831389512	2.316454959	1.855404429
8.00	8.573184508	2.388423484	1.882153976
9.00	9.295086900	2.450953928	1.904497853
10.00	10.00000000	2.506184146	1.923584036
11.00	10.69015604	2.555604612	1.940175270
12.00	11.36731780	2.600295000	1.954801771
13.00	12.03290801	2.641061916	1.967845700
14.00	12.68809590	2.678523486	1.979590786
15.00	13.33385721	2.713163604	1.990252953
16.00	13.97101690	2.745368024	2.000000000
17.00	14.60028043	2.775449105	2.008964691
18.00	15.22225700	2.803663246	2.017253720
19.00	15.83747737	2.830223438	2.024954010
20.00	16.44640751	2.855308503	2.032137232
21.00	17.04945935	2.879069993	2.038863121
22.00	17.64699933	2.901637447	2.045181942
23.00	18.23935530	2.923122436	2.051136359
24.00	18.82682201	2.943621727	2.056762871
25.00	19.40966577	2.963219775	2.062092922
26.00	19.98812819	2.981990722	2.067153779
27.00	20.56242932	3.000000000	2.071969229
28.00	21.13277033	3.017305639	2.076560138
29.00	21.69933575	3.033959336	2.080944900
30.00	22.26229534	3.050007342	2.085139807
31.00	22.82180578	3.065491199	2.089159355
32.00	23.37801202	3.080448350	2.093016489
33.00	23.93104852	3.094912663	2.096722817
34.00	24.48104031	3.108914870	2.100288785
35.00	25.02810389	3.122482939	2.103723821
36.00	25.57234809	3.135642394	2.107036464
37.00	26.11387469	3.148416593	2.110234471
38.00	26.65277915	3.160826963	2.113324904
39.00	27.18915109	3.172893208	2.116314215
40.00	27.72307482	3.184633484	2.119208308
41.00	28.25462977	3.196064562	2.122012601
42.00	28.78389089	3.207201961	2.124732076
43.00	29.31092902	3.218060071	2.127371325
44.00	29.83581116	3.228652256	2.129934589
45.00	30.35860082	3.238990953	2.132425789
46.00	30.87935822	3.249087754	2.134848561
47.00	31.39814057	3.258953479	2.137206281
48.00	31.91500227	3.268598245	2.139502088
49.00	32.42999509	3.278031524	2.141738907
50.00	32.94316837	3.287262195	2.143919465
51.00	33.45456916	3.296298598	2.146046313
52.00	33.96424240	3.305148568	2.148121833
53.00	34.47223099	3.313819482	2.150148261
54.00	34.97857602	3.322318291	2.152127692
55.00	35.48331678	3.330651551	2.154062094
56.00	35.98649092	3.338825455	2.155953319
57.00	36.48813456	3.346845857	2.157803108
58.00	36.98828233	3.354718297	2.159613104
59.00	37.48696749	3.362448022	2.161384857

60.00	37.98422201	3.370040008	2.163119828
61.00	38.48007663	3.377498976	2.164819403
62.00	38.97456092	3.384829413	2.166484890
63.00	39.46770334	3.392035582	2.168117528
64.00	39.95953134	3.399121540	2.169718494
65.00	40.45007134	3.406091150	2.171288902
66.00	40.93934887	3.412948093	2.172829812
67.00	41.42738853	3.419695880	2.174342230
68.00	41.91421409	3.426337861	2.175827114
69.00	42.39984851	3.432877236	2.177285376
70.00	42.88431399	3.439317062	2.178717883
71.00	43.36763200	3.445660265	2.180125463
72.00	43.84982331	3.451909642	2.181508906
73.00	44.33090801	3.458067873	2.182868966
74.00	44.81090559	3.464137524	2.184206362
75.00	45.28983490	3.470121057	2.185521783
76.00	45.76771424	3.476020832	2.186815887
77.00	46.24456133	3.481839116	2.188089304
78.00	46.72039338	3.487578084	2.189342637
79.00	47.19522708	3.493239826	2.190576464
80.00	47.66907865	3.498826352	2.191791339
81.00	48.14196382	3.504339594	2.192987795
82.00	48.61389788	3.509781412	2.194166340
83.00	49.08489572	3.515153596	2.195327465
84.00	49.55497178	3.520457872	2.196471640
85.00	50.02414012	3.525695900	2.197599318
86.00	50.49241441	3.530869282	2.198710934
87.00	50.95980798	3.535979563	2.199806906
88.00	51.42633379	3.541028234	2.200887636
89.00	51.89200445	3.546016733	2.201953514
90.00	52.35683226	3.550946450	2.203004911
91.00	52.82082921	3.555818726	2.204042189
92.00	53.28400697	3.560634860	2.205065695
93.00	53.74637694	3.565396105	2.206075764
94.00	54.20795022	3.570103673	2.207072719
95.00	54.66873765	3.574758738	2.208056872
96.00	55.12874980	3.579362435	2.209028524
97.00	55.58799700	3.583915864	2.209987967
98.00	56.04648931	3.588420090	2.210935482
99.00	56.50423659	3.592876143	2.211871341
100.00	56.96124843	3.597285024	2.212795806
101.00	57.41753424	3.601647701	2.213709134
102.00	57.87310320	3.605965114	2.214611569
103.00	58.32796426	3.610238175	2.215503351
104.00	58.78212622	3.614467768	2.216384710
105.00	59.23559763	3.618654753	2.217255870
106.00	59.68838691	3.622799961	2.218117048
107.00	60.14050224	3.626904203	2.218968453
108.00	60.59195168	3.630968265	2.219810291
109.00	61.04274307	3.634992912	2.220642759
110.00	61.49288412	3.638978885	2.221466047
111.00	61.94238237	3.642926907	2.222280343
112.00	62.39124519	3.646837680	2.223085827
113.00	62.83947982	3.650711887	2.223882674
114.00	63.28709333	3.654550193	2.224671054
115.00	63.73409267	3.658353244	2.225451135
116.00	64.18048465	3.662121669	2.226223075
117.00	64.62627592	3.665856082	2.226987031
118.00	65.07147303	3.669557079	2.227743156
119.00	65.51608239	3.673225243	2.228491598
120.00	65.96011028	3.676861138	2.229232499
121.00	66.40356287	3.680465319	2.229966000

122.00	66.84644622	3.684038321	2.230692238
123.00	67.28876625	3.687580671	2.231411344
124.00	67.73052879	3.691092880	2.232123448
125.00	68.17173957	3.694575447	2.232828675
126.00	68.61240419	3.698028858	2.233527148
127.00	69.05252816	3.701453589	2.234218986
128.00	69.49211691	3.704850104	2.234904305
129.00	69.93117574	3.708218854	2.235583218
130.00	70.36970988	3.711560281	2.236255837
131.00	70.80772445	3.714874818	2.236922268
132.00	71.24522449	3.718162885	2.237582617
133.00	71.68221496	3.721424893	2.238236985
134.00	72.11870073	3.724661246	2.238885474
135.00	72.55468656	3.727872335	2.239528180
136.00	72.99017718	3.731058546	2.240165199
137.00	73.42517720	3.734220253	2.240796623
138.00	73.85969117	3.737357823	2.241422543
139.00	74.29372356	3.740471615	2.242043049
140.00	74.72727876	3.743561982	2.242658225
141.00	75.16036111	3.746629264	2.243268158
142.00	75.59297485	3.749673800	2.243872929
143.00	76.02512418	3.752695917	2.244472619
144.00	76.45681321	3.755695937	2.245067307
145.00	76.88804599	3.758674176	2.245657070
146.00	77.31882653	3.761630940	2.246241984
147.00	77.74915874	3.764566533	2.246822121
148.00	78.17904650	3.767481249	2.247397556
149.00	78.60849360	3.770375378	2.247968356
150.00	79.03750380	3.773249203	2.248534593
151.00	79.46608079	3.776103002	2.249096333
152.00	79.89422821	3.778937048	2.249653642
153.00	80.32194963	3.781751606	2.250206585
154.00	80.74924859	3.784546939	2.250755225
155.00	81.17612856	3.787323301	2.251299624
156.00	81.60259295	3.790080945	2.251839843
157.00	82.02864516	3.792820117	2.252375941
158.00	82.45428849	3.795541057	2.252907977
159.00	82.87952623	3.798244002	2.253436007
160.00	83.30436161	3.800929185	2.253960087
161.00	83.72879781	3.803596833	2.254480271
162.00	84.15283797	3.806247170	2.254996614
163.00	84.57648518	3.808880415	2.255509169
164.00	84.99974249	3.811496783	2.256017985
165.00	85.42261291	3.814096485	2.256523114
166.00	85.84509941	3.816679727	2.257024606
167.00	86.26720491	3.819246715	2.257522509
168.00	86.68893230	3.821797646	2.258016870
169.00	87.11028442	3.824332718	2.258507736
170.00	87.53126409	3.826852122	2.258995153
171.00	87.95187407	3.829356047	2.259479166
172.00	88.37211709	3.831844680	2.259959819
173.00	88.79199586	3.834318203	2.260437155
174.00	89.21151304	3.836776794	2.260911216
175.00	89.63067124	3.839220629	2.261382045
176.00	90.04947307	3.841649882	2.261849681
177.00	90.46792107	3.844064722	2.262314166
178.00	90.88601779	3.846465317	2.262775538
179.00	91.30376570	3.848851830	2.263233836
180.00	91.72116728	3.851224423	2.263689099
181.00	92.13822495	3.853583254	2.264141363
182.00	92.55494111	3.855928480	2.264590665
183.00	92.97131814	3.858260252	2.265037041

184.00	93.38735836	3.860578723	2.265480526
185.00	93.80306410	3.862884041	2.265921156
186.00	94.21843763	3.865176350	2.266358964
187.00	94.63348121	3.867455795	2.266793983
188.00	95.04819706	3.869722516	2.267226248
189.00	95.46258740	3.871976653	2.267655789
190.00	95.87665438	3.874218342	2.268082639
191.00	96.29040016	3.876447716	2.268506830
192.00	96.70382686	3.878664909	2.268928391
193.00	97.11693657	3.880870050	2.269347354
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195.00	97.94221330	3.885244688	2.270177601
196.00	98.35438440	3.887414434	2.270588944
197.00	98.76624665	3.889572628	2.270997804
198.00	99.17780203	3.891719391	2.271404209
199.00	99.58905251	3.893854841	2.271808187
200.00	100.0000000	3.895979094	2.272209764
201.00	100.4106464	3.898092266	2.272608967
202.00	100.8209937	3.900194468	2.273005821
203.00	101.2310436	3.902285814	2.273400354
204.00	101.6407980	3.904366411	2.273792589
205.00	102.0502589	3.906436368	2.274182552
206.00	102.4594278	3.908495792	2.274570267
207.00	102.8683068	3.910544786	2.274955758
208.00	103.2768974	3.912583456	2.275339049
209.00	103.6852015	3.914611901	2.275720163
210.00	104.0932207	3.916630223	2.276099124
211.00	104.5009569	3.918638520	2.276475953
212.00	104.9084116	3.920636890	2.276850674
213.00	105.3155866	3.922625428	2.277223308
214.00	105.7224835	3.924604230	2.277593876
215.00	106.1291040	3.926573387	2.277962400
216.00	106.5354496	3.928532993	2.278328901
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218.00	107.3473226	3.932423911	2.279055917
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220.00	108.1581154	3.936277693	2.279775083
221.00	108.5631105	3.938190874	2.280131773
222.00	108.9678401	3.940095029	2.280486558
223.00	109.3723057	3.941990240	2.280839459
224.00	109.7765089	3.943876590	2.281190494
225.00	110.1804511	3.945754160	2.281539681
226.00	110.5841337	3.947623029	2.281887038
227.00	110.9875582	3.949483277	2.282232584
228.00	111.3907260	3.951334982	2.282576335
229.00	111.7936386	3.953178219	2.282918309
230.00	112.1962973	3.955013066	2.283258523
231.00	112.5987035	3.956839596	2.283596994
232.00	113.0008587	3.958657884	2.283933739
233.00	113.4027641	3.960468002	2.284268774
234.00	113.8044211	3.962270021	2.284602115
235.00	114.2058311	3.964064014	2.284933778
236.00	114.6069953	3.965850050	2.285263779
237.00	115.0079152	3.967628197	2.285592133
238.00	115.4085918	3.969398525	2.285918856
239.00	115.8090267	3.971161100	2.286243962
240.00	116.2092209	3.972915989	2.286567467
241.00	116.6091759	3.974663258	2.286889385
242.00	117.0088928	3.976402971	2.287209731
243.00	117.4083728	3.978135192	2.287528520
244.00	117.8076173	3.979859984	2.287845764
245.00	118.2066274	3.981577410	2.288161478

246.00	118.6054042	3.983287531	2.288475677
247.00	119.0039491	3.984990408	2.288788372
248.00	119.4022631	3.986686102	2.289099579
249.00	119.8003475	3.988374671	2.289409309
250.00	120.1982034	3.990056174	2.289717577
251.00	120.5958319	3.991730669	2.290024394
252.00	120.9932343	3.993398213	2.290329774
253.00	121.3904115	3.995058862	2.290633729
254.00	121.7873648	3.996712674	2.290936271
255.00	122.1840952	3.998359701	2.291237414
256.00	122.5806039	4.000000000	2.291537168
257.00	122.9768918	4.001633624	2.291835546
258.00	123.3729602	4.003260625	2.292132560
259.00	123.7688100	4.004881058	2.292428220
260.00	124.1644424	4.006494973	2.292722540
261.00	124.5598583	4.008102421	2.293015529
262.00	124.9550589	4.009703455	2.293307199
263.00	125.3500451	4.011298123	2.293597562
264.00	125.7448180	4.012886475	2.293886628
265.00	126.1393785	4.014468561	2.294174408
266.00	126.5337278	4.016044429	2.294460912
267.00	126.9278667	4.017614126	2.294746152
268.00	127.3217963	4.019177700	2.295030138
269.00	127.7155175	4.020735197	2.295312879
270.00	128.1090314	4.022286665	2.295594386
271.00	128.5023388	4.023832148	2.295874669
272.00	128.8954408	4.025371692	2.296153739
273.00	129.2883383	4.026905342	2.296431604
274.00	129.6810322	4.028433141	2.296708274
275.00	130.0735234	4.029955134	2.296983760
276.00	130.4658130	4.031471364	2.297258070
277.00	130.8579017	4.032981873	2.297531214
278.00	131.2497905	4.034486704	2.297803202
279.00	131.6414803	4.035985899	2.298074041
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286.00	134.3778114	4.046325859	2.299938527
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291.00	136.3265548	4.053551208	2.301237780
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293.00	137.1047310	4.056405165	2.301750160
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296.00	138.2705998	4.060648235	2.302511080
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305.00	141.7583552	4.073113432	2.304740632
306.00	142.1449878	4.074474722	2.304983588
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323.00	148.6913139	4.096932619	2.308976851
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410.00	181.5051581	4.195400420	2.326161761
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412.00	182.2475292	4.197399490	2.326505313
413.00	182.6185320	4.198395243	2.326676362
414.00	182.9894133	4.199388491	2.326846929
415.00	183.3601737	4.200379246	2.327017017
416.00	183.7308134	4.201367522	2.327186628
417.00	184.1013327	4.202353330	2.327355765
418.00	184.4717322	4.203336681	2.327524431
419.00	184.8420121	4.204317589	2.327692627
420.00	185.2121727	4.205296066	2.327860357
421.00	185.5822145	4.206272122	2.328027622
422.00	185.9521378	4.207245770	2.328194426
423.00	186.3219428	4.208217021	2.328360770
424.00	186.6916301	4.209185887	2.328526657
425.00	187.0611999	4.210152380	2.328692089
426.00	187.4306525	4.211116510	2.328857069
427.00	187.7999884	4.212078290	2.329021599
428.00	188.1692078	4.213037730	2.329185681
429.00	188.5383111	4.213994842	2.329349318
430.00	188.9072986	4.214949636	2.329512511
431.00	189.2761707	4.215902125	2.329675263

432.00	189.6449276	4.216852318	2.329837577
433.00	190.0135698	4.217800226	2.329999454
434.00	190.3820976	4.218745861	2.330160896
435.00	190.7505112	4.219689233	2.330321907
436.00	191.1188110	4.220630352	2.330482487
437.00	191.4869973	4.221569230	2.330642640
438.00	191.8550705	4.222505877	2.330802367
439.00	192.2230309	4.223440302	2.330961671
440.00	192.5908788	4.224372518	2.331120553
441.00	192.9586144	4.225302533	2.331279015
442.00	193.3262382	4.226230357	2.331437060
443.00	193.6937505	4.227156002	2.331594690
444.00	194.0611515	4.228079477	2.331751907
445.00	194.4284415	4.229000793	2.331908713
446.00	194.7956210	4.229919958	2.332065109
447.00	195.1626901	4.230836983	2.332221099
448.00	195.5296492	4.231751878	2.332376683
449.00	195.8964986	4.232664652	2.332531864
450.00	196.2632386	4.233575315	2.332686644
451.00	196.6298695	4.234483877	2.332841024
452.00	196.9963916	4.235390347	2.332995007
453.00	197.3628052	4.236294735	2.333148595
454.00	197.7291105	4.237197049	2.333301789
455.00	198.0953080	4.238097300	2.333454592
456.00	198.4613979	4.238995497	2.333607004
457.00	198.8273804	4.239891649	2.333759029
458.00	199.1932560	4.240785764	2.333910668
459.00	199.5590247	4.241677852	2.334061922
460.00	199.9246871	4.242567923	2.334212795
461.00	200.2902432	4.243455985	2.334363286
462.00	200.6556935	4.244342046	2.334513399
463.00	201.0210382	4.245226117	2.334663135
464.00	201.3862776	4.246108205	2.334812495
465.00	201.7514120	4.246988319	2.334961483
466.00	202.1164416	4.247866468	2.335110098
467.00	202.4813668	4.248742661	2.335258344
468.00	202.8461877	4.249616906	2.335406221
469.00	203.2109048	4.250489212	2.335553732
470.00	203.5755182	4.251359587	2.335700878
471.00	203.9400282	4.252228040	2.335847660
472.00	204.3044352	4.253094578	2.335994082
473.00	204.6687393	4.253959211	2.336140143
474.00	205.0329409	4.254821945	2.336285847
475.00	205.3970402	4.255682791	2.336431194
476.00	205.7610375	4.256541755	2.336576186
477.00	206.1249330	4.257398846	2.336720825
478.00	206.4887271	4.258254072	2.336865112
479.00	206.8524199	4.259107440	2.337009050
480.00	207.2160118	4.259958960	2.337152639
481.00	207.5795029	4.260808637	2.337295881
482.00	207.9428937	4.261656482	2.337438778
483.00	208.3061842	4.262502500	2.337581331
484.00	208.6693748	4.263346700	2.337723543
485.00	209.0324658	4.264189090	2.337865413
486.00	209.3954573	4.265029677	2.338006945
487.00	209.7583497	4.265868469	2.338148139
488.00	210.1211432	4.266705473	2.338288997
489.00	210.4838380	4.267540697	2.338429520
490.00	210.8464345	4.268374148	2.338569710
491.00	211.2089327	4.269205834	2.338709569
492.00	211.5713331	4.270035762	2.338849098
493.00	211.9336358	4.270863939	2.338988298

494.00	212.2958411	4.271690372	2.339127171
495.00	212.6579492	4.272515070	2.339265718
496.00	213.0199604	4.273338038	2.339403941
497.00	213.3818749	4.274159284	2.339541841
498.00	213.7436930	4.274978816	2.339679420
499.00	214.1054148	4.275796639	2.339816679
500.00	214.4670407	4.276612762	2.339953618
501.00	214.8285709	4.277427191	2.340090241
502.00	215.1900056	4.278239932	2.340226548
503.00	215.5513450	4.279050994	2.340362540
504.00	215.9125894	4.279860382	2.340498219
505.00	216.2737390	4.280668104	2.340633587
506.00	216.6347941	4.281474166	2.340768643
507.00	216.9957549	4.282278575	2.340903391
508.00	217.3566215	4.283081338	2.341037831
509.00	217.7173943	4.283882460	2.341171965
510.00	218.0780735	4.284681950	2.341305793
511.00	218.4386592	4.285479813	2.341439317
512.00	218.7991518	4.286276055	2.341572539
513.00	219.1595515	4.287070684	2.341705460
514.00	219.5198584	4.287863706	2.341838081
515.00	219.8800728	4.288655127	2.341970402
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517.00	220.6002250	4.290233191	2.342234155
518.00	220.9601632	4.291019847	2.342365588
519.00	221.3200098	4.291804927	2.342496728
520.00	221.6797650	4.292588438	2.342627575
521.00	222.0394290	4.293370386	2.342758131
522.00	222.3990021	4.294150776	2.342888397
523.00	222.7584844	4.294929615	2.343018374
524.00	223.1178761	4.295706909	2.343148063
525.00	223.4771776	4.296482664	2.343277466
526.00	223.8363889	4.297256887	2.343406584
527.00	224.1955103	4.298029581	2.343535417
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529.00	224.9134844	4.299570414	2.343792237
530.00	225.2723374	4.300338563	2.343920225
531.00	225.6311013	4.301105208	2.344047934
532.00	225.9897765	4.301870356	2.344175365
533.00	226.3483629	4.302634011	2.344302519
534.00	226.7068610	4.303396181	2.344429396
535.00	227.0652708	4.304156869	2.344555999
536.00	227.4235925	4.304916083	2.344682328
537.00	227.7818265	4.305673827	2.344808384
538.00	228.1399728	4.306430107	2.344934169
539.00	228.4980317	4.307184929	2.345059683
540.00	228.8560034	4.307938298	2.345184928
541.00	229.2138881	4.308690220	2.345309905
542.00	229.5716860	4.309440700	2.345434614
543.00	229.9293972	4.310189744	2.345559058
544.00	230.2870220	4.310937357	2.345683236
545.00	230.6445606	4.311683545	2.345807150
546.00	231.0020132	4.312428312	2.345930802
547.00	231.3593799	4.313171664	2.346054191
548.00	231.7166610	4.313913607	2.346177320
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552.00	233.1449327	4.316867386	2.346667247
553.00	233.5017885	4.317602359	2.346789087
554.00	233.8585598	4.318335953	2.346910672
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562.00	236.7097106	4.324155685	2.347874303
563.00	237.0657299	4.324877098	2.347993640
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567.00	238.4889822	4.327749509	2.348468548
568.00	238.8445901	4.328464325	2.348586670
569.00	239.2001162	4.329177836	2.348704551
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573.00	240.6214068	4.332018922	2.349173692
574.00	240.9765268	4.332725977	2.349290385
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577.00	242.0414036	4.334839497	2.349639060
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581.00	243.4601178	4.337639849	2.350100713
582.00	243.8145972	4.338336811	2.350215552
583.00	244.1689973	4.339032531	2.350330163
584.00	244.5233182	4.339727013	2.350444548
585.00	244.8775602	4.340420263	2.350558706
586.00	245.2317234	4.341112284	2.350672639
587.00	245.5858079	4.341803080	2.350786347
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589.00	246.2937416	4.343181016	2.351013093
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591.00	247.0013628	4.344554105	2.351238951
592.00	247.3550565	4.345238842	2.351351549
593.00	247.7086726	4.345922380	2.351463927
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597.00	249.1223636	4.348644621	2.351911261
598.00	249.4755938	4.349322224	2.352022553
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600.00	250.1818243	4.350673910	2.352244494
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607.00	252.6512331	4.355368263	2.353014609
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611.00	254.0606640	4.358025596	2.353450084
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629.00	260.3884519	4.369764382	2.355369782
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631.00	261.0900830	4.371047125	2.355579160
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643.00	265.2938795	4.378656104	2.356819552
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737.00	297.8911968	4.433607245	2.365697399
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.46	1.082620621	7.5135763759E-02	2.534632205	1.020675594
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.48	1.157130486	8.0504441902E-02	2.611881570	.9972273891

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.53	1.336065569	9.4479578895E-02	2.796647074	.9440779077
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.58	1.505850267	.1092071260	2.971070911	.8974782548
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3.03	6.586400297	1.334606103	7.951250586	.3149567115
3.04	6.602445640	1.340838441	7.966365971	.3142384802
3.05	6.618474061	1.347077359	7.981461672	.3135240282
3.06	6.634485658	1.353322833	7.996537794	.3128133241
3.07	6.650480528	1.359574838	8.011594442	.3121063367
3.08	6.666458770	1.365833350	8.026631719	.3114030353
3.09	6.682420478	1.372098344	8.041649728	.3107033896
3.10	6.698365748	1.378369797	8.056648571	.3100073694
3.11	6.714294675	1.384647684	8.071628348	.3093149451
3.12	6.730207353	1.390931982	8.086589160	.3086260872
3.13	6.746103873	1.397222667	8.101531106	.3079407668
3.14	6.761984328	1.403519717	8.116454284	.3072589551
3.15	6.777848810	1.409823106	8.131358791	.3065806236
3.16	6.793697407	1.416132813	8.146244725	.3059057444
3.17	6.809530211	1.422448815	8.161112180	.3052342896
3.18	6.825347310	1.428771087	8.175961252	.3045662317
3.19	6.841148792	1.435099608	8.190792035	.3039015437
3.20	6.856934743	1.441434355	8.205604623	.3032401985
3.21	6.872705251	1.447775305	8.220399107	.3025821696
3.22	6.888460402	1.454122436	8.235175580	.3019274308
3.23	6.904200280	1.460475726	8.249934133	.3012759560
3.24	6.919924970	1.466835153	8.264674856	.3006277196
3.25	6.935634555	1.473200694	8.279397838	.2999826959
3.26	6.951329118	1.479572327	8.294103169	.2993408600
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3.28	6.982673507	1.492333786	8.323461227	.2980666518
3.29	6.998323496	1.498723567	8.338114128	.2974342306
3.30	7.013958787	1.505119355	8.352749725	.2968048991
3.31	7.029579461	1.511521128	8.367368103	.2961786333
3.32	7.045185595	1.517928865	8.381969348	.2955554098
3.33	7.060777269	1.524342545	8.396553542	.2949352051
3.34	7.076354560	1.530762147	8.411120769	.2943179962
3.35	7.091917545	1.537187651	8.425671111	.2937037601
3.36	7.107466299	1.543619034	8.440204650	.2930924743
3.37	7.123000899	1.550056278	8.454721467	.2924841163
3.38	7.138521420	1.556499361	8.469221643	.2918786640
3.39	7.154027935	1.562948264	8.483705258	.2912760955
3.40	7.169520519	1.569402965	8.498172390	.2906763890
3.41	7.184999246	1.575863446	8.512623119	.2900795231
3.42	7.200464186	1.582329685	8.527057523	.2894854766
3.43	7.215915413	1.588801663	8.541475678	.2888942283
3.44	7.231352998	1.595279361	8.555877661	.2883057574
3.45	7.246777012	1.601762758	8.570263549	.2877200435
3.46	7.262187525	1.608251835	8.584633418	.2871370659
3.47	7.277584607	1.614746572	8.598987341	.2865568046
3.48	7.292968327	1.621246951	8.613325395	.2859792396
3.49	7.308338754	1.627752952	8.627647651	.2854043510
3.50	7.323695955	1.634264556	8.641954185	.2848321192
3.51	7.339039999	1.640781743	8.656245067	.2842625250
3.52	7.354370952	1.647304495	8.670520371	.2836955490
3.53	7.369688880	1.653832793	8.684780169	.2831311723
3.54	7.384993851	1.660366618	8.699024530	.2825693760
3.55	7.400285929	1.666905952	8.713253526	.2820101415
3.56	7.415565179	1.673450777	8.727467227	.2814534504
3.57	7.430831666	1.680001072	8.741665702	.2808992843
3.58	7.446085454	1.686556822	8.755849020	.2803476253

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3.60	7.476555185	1.699684607	8.784170457	.2792517567
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3.62	7.506974873	1.712833989	8.812432081	.2781657034
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3.65	7.552511657	1.732598240	8.854713530	.2765547251
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3.67	7.582808388	1.745800987	8.882827935	.2754926102
3.68	7.597938595	1.752410283	8.896863362	.2749650642
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3.72	7.658339732	1.778899872	8.952861381	.2728779127
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3.74	7.688469294	1.792175873	8.980775048	.2718479348
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3.77	7.733576151	1.812128502	9.022540247	.2703196173
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3.79	7.763589712	1.825455814	9.050314211	.2693116628
3.80	7.778579353	1.832127088	9.064180520	.2688109183
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3.82	7.808524625	1.845484792	9.091872083	.2678158223
3.83	7.823480363	1.852171189	9.105697452	.2673214423
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3.86	7.868280616	1.872260370	9.147092595	.2658508045
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3.89	7.912981511	1.892394203	9.188367469	.2643986309
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3.92	7.957584422	1.912572266	9.229523560	.2629645574
3.93	7.972430518	1.919308044	9.243216114	.2624904936
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3.95	8.002090692	1.932794144	9.270562328	.2615482297
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4.16	8.311035747	2.075540448	9.554665082	.2521029648
4.17	8.325638102	2.082388659	9.568059965	.2516726051
4.18	8.340230849	2.089241379	9.581443042	.2512439268
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4.20	8.369387686	2.102960290	9.608173961	.2503915727

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4.36	8.601303274	2.213346223	9.820369834	.2438038239
4.37	8.615721037	2.220282112	9.833536873	.2434052179
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8.21	13.71621129	5.115952664	14.32212846	.1535242360
8.22	13.72872241	5.123945552	14.33275532	.1533847983
8.23	13.74123075	5.131940220	14.34337809	.1532456460
8.24	13.75373631	5.139936663	14.35399679	.1531067782
8.25	13.76623910	5.147934879	14.36461142	.1529681940
8.26	13.77873912	5.155934864	14.37522200	.1528298924
8.27	13.79123638	5.163936616	14.38582852	.1526918726
8.28	13.80373089	5.171940131	14.39643100	.1525541337
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8.30	13.82871168	5.187952439	14.41762387	.1522794950
8.31	13.84119796	5.195961225	14.42821426	.1521425933
8.32	13.85368152	5.203971761	14.43880064	.1520059690
8.33	13.86616236	5.211984045	14.44938302	.1518696211
8.34	13.87864048	5.219998074	14.45996140	.1517335487
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8.37	13.91605859	5.244050594	14.49167261	.1513269763
8.38	13.92852591	5.252071569	14.50223506	.1511919975
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8.47	14.04061153	5.324337777	14.59711971	.1499892915
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8.54	14.12764150	5.380640228	14.67070031	.1490686508

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9.14	14.86858881	5.866510405	15.29388760	.1416693236
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15.88	22.78193231	11.60993362	21.63596475	9.2937316931E-02
15.89	22.79329318	11.61873576	21.64471399	9.2892021376E-02
15.90	22.80465324	11.62753853	21.65346171	9.2846774959E-02
15.91	22.81601250	11.63634192	21.66220791	9.2801577594E-02
15.92	22.82737095	11.64514594	21.67095260	9.2756429199E-02
15.93	22.83872859	11.65395058	21.67969578	9.2711329691E-02
15.94	22.85008543	11.66275585	21.68843745	9.2666278989E-02
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15.97	22.88415112	11.68917539	21.71465340	9.2531418885E-02
15.98	22.89550475	11.69798315	21.72338905	9.2486562578E-02

15.99	22.90685757	11.70679153	21.73212318	9.2441754666E-02
16.00	22.91820960	11.71560053	21.74085582	9.2396995068E-02
16.01	22.92956083	11.72441015	21.74958696	9.2352283701E-02
16.02	22.94091126	11.73322038	21.75831659	9.2307620484E-02
16.03	22.95226090	11.74203124	21.76704473	9.2263005337E-02
16.04	22.96360974	11.75084271	21.77577137	9.2218438180E-02
16.05	22.97495779	11.75965480	21.78449652	9.2173918931E-02
16.06	22.98630504	11.76846751	21.79322017	9.2129447510E-02
16.07	22.99765150	11.77728083	21.80194233	9.2085023837E-02
16.08	23.00899717	11.78609477	21.81066300	9.2040647831E-02
16.09	23.02034205	11.79490932	21.81938219	9.1996319414E-02
16.10	23.03168613	11.80372448	21.82809988	9.1952038506E-02
16.11	23.04302943	11.81254026	21.83681608	9.1907805027E-02
16.12	23.05437194	11.82135665	21.84553080	9.1863618897E-02
16.13	23.06571367	11.83017365	21.85424404	9.1819480039E-02
16.14	23.07705461	11.83899126	21.86295579	9.1775388372E-02
16.15	23.08839476	11.84780949	21.87166607	9.1731343818E-02
16.16	23.09973413	11.85662832	21.88037486	9.1687346299E-02
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16.18	23.12241052	11.87426781	21.89778801	9.1599492051E-02
16.19	23.13374754	11.88308847	21.90649237	9.1555635166E-02
16.20	23.14508378	11.89190974	21.91519525	9.1511825003E-02
16.21	23.15641924	11.90073161	21.92389666	9.1468061485E-02
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16.23	23.17908783	11.91837717	21.94129507	9.1380674072E-02
16.24	23.19042096	11.92720086	21.94999207	9.1337050023E-02
16.25	23.20175331	11.93602516	21.95868761	9.1293472310E-02
16.26	23.21308489	11.94485005	21.96738167	9.1249940855E-02
16.27	23.22441569	11.95367555	21.97607427	9.1206455583E-02
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16.35	23.31503435	12.02430118	22.04556248	9.0860226829E-02
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16.56	23.55267623	12.20987457	22.22752817	8.9965133704E-02
16.57	23.56398422	12.21871787	22.23617741	8.9922999113E-02
16.58	23.57529146	12.22756176	22.24482524	8.9880908414E-02
16.59	23.58659796	12.23640623	22.25347164	8.9838861533E-02
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16.63	23.63181651	12.27178994	22.28804302	8.9671110782E-02
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16.66	23.66572264	12.29833382	22.31395667	8.9545754349E-02
16.67	23.67702320	12.30718293	22.32259173	8.9504055467E-02
16.68	23.68832302	12.31603263	22.33122537	8.9462399776E-02
16.69	23.69962211	12.32488290	22.33985760	8.9420787199E-02
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16.71	23.72221807	12.34258518	22.35711783	8.9337691112E-02
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16.78	23.80128083	12.40456126	22.41748436	8.9048202733E-02
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16.83	23.85773233	12.44884706	22.46056132	8.8842699458E-02
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17.01	24.06080841	12.60839320	22.61535113	8.8111539702E-02
17.02	24.07208362	12.61726222	22.62393748	8.8071311753E-02
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17.23	24.30870032	12.80363962	22.80393600	8.7235858297E-02
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18.21	25.40897877	13.67651197	23.63624824	8.3558643449E-02
18.22	25.42017413	13.68544424	23.64467815	8.3522889938E-02
18.23	25.43136887	13.69437701	23.65310682	8.3487170399E-02
18.24	25.44256298	13.70331028	23.66153425	8.3451484782E-02
18.25	25.45375647	13.71224404	23.66996045	8.3415833036E-02
18.26	25.46494934	13.72117830	23.67838540	8.3380215110E-02
18.27	25.47614159	13.73011305	23.68680913	8.3344630956E-02
18.28	25.48733322	13.73904830	23.69523161	8.3309080523E-02
18.29	25.49852423	13.74798405	23.70365287	8.3273563761E-02
18.30	25.50971462	13.75692029	23.71207289	8.3238080621E-02
18.31	25.52090439	13.76585702	23.72049168	8.3202631053E-02
18.32	25.53209355	13.77479424	23.72890923	8.3167215007E-02
18.33	25.54328208	13.78373196	23.73732556	8.3131832434E-02
18.34	25.55447001	13.79267017	23.74574066	8.3096483285E-02
18.35	25.56565731	13.80160887	23.75415453	8.3061167510E-02
18.36	25.57684401	13.81054807	23.76256718	8.3025885060E-02
18.37	25.58803008	13.81948775	23.77097859	8.2990635887E-02
18.38	25.59921555	13.82842793	23.77938879	8.2955419942E-02
18.39	25.61040040	13.83736859	23.78779776	8.2920237175E-02
18.40	25.62158464	13.84630974	23.79620550	8.2885087537E-02
18.41	25.63276826	13.85525139	23.80461203	8.2849970981E-02
18.42	25.64395128	13.86419352	23.81301733	8.2814887458E-02
18.43	25.65513369	13.87313614	23.82142141	8.2779836919E-02
18.44	25.66631548	13.88207924	23.82982428	8.2744819315E-02
18.45	25.67749667	13.89102284	23.83822592	8.2709834600E-02
18.46	25.68867725	13.89996692	23.84662635	8.2674882724E-02

18.47	25.69985722	13.90891149	23.85502556	8.2639963639E-02
18.48	25.71103658	13.91785654	23.86342356	8.2605077298E-02
18.49	25.72221534	13.92680208	23.87182034	8.2570223652E-02
18.50	25.73339349	13.93574810	23.88021591	8.2535402654E-02
18.51	25.74457103	13.94469461	23.88861027	8.2500614257E-02
18.52	25.75574797	13.95364160	23.89700341	8.2465858412E-02
18.53	25.76692431	13.96258908	23.90539535	8.2431135072E-02
18.54	25.77810004	13.97153704	23.91378607	8.2396444190E-02
18.55	25.78927517	13.98048548	23.92217559	8.2361785718E-02
18.56	25.80044970	13.98943440	23.93056390	8.2327159610E-02
18.57	25.81162363	13.99838381	23.93895100	8.2292565817E-02
18.58	25.82279695	14.00733370	23.94733690	8.2258004294E-02
18.59	25.83396968	14.01628407	23.95572159	8.2223474993E-02
18.60	25.84514180	14.02523491	23.96410508	8.2188977868E-02
18.61	25.85631333	14.03418624	23.97248736	8.2154512871E-02
18.62	25.86748426	14.04313805	23.98086845	8.2120079955E-02
18.63	25.87865459	14.05209034	23.98924833	8.2085679076E-02
18.64	25.88982432	14.06104311	23.99762701	8.2051310185E-02
18.65	25.90099345	14.06999635	24.00600450	8.2016973237E-02
18.66	25.91216199	14.07895007	24.01438078	8.1982668185E-02
18.67	25.92332993	14.08790427	24.02275587	8.1948394984E-02
18.68	25.93449728	14.09685895	24.03112976	8.1914153586E-02
18.69	25.94566404	14.10581410	24.03950246	8.1879943947E-02
18.70	25.95683020	14.11476973	24.04787396	8.1845766020E-02
18.71	25.96799577	14.12372584	24.05624427	8.1811619759E-02
18.72	25.97916074	14.13268242	24.06461338	8.1777505118E-02
18.73	25.99032513	14.14163947	24.07298130	8.1743422053E-02
18.74	26.00148892	14.15059700	24.08134804	8.1709370516E-02
18.75	26.01265212	14.15955501	24.08971358	8.1675350464E-02
18.76	26.02381473	14.16851348	24.09807794	8.1641361851E-02
18.77	26.03497675	14.17747243	24.10644110	8.1607404630E-02
18.78	26.04613819	14.18643186	24.11480308	8.1573478758E-02
18.79	26.05729903	14.19539175	24.12316388	8.1539584188E-02
18.80	26.06845929	14.20435212	24.13152348	8.1505720877E-02
18.81	26.07961896	14.21331296	24.13988191	8.1471888778E-02
18.82	26.09077804	14.22227427	24.14823915	8.1438087848E-02
18.83	26.10193654	14.23123605	24.15659521	8.1404318041E-02
18.84	26.11309445	14.24019830	24.16495009	8.1370579313E-02
18.85	26.12425178	14.24916102	24.17330378	8.1336871619E-02
18.86	26.13540852	14.25812420	24.18165630	8.1303194915E-02
18.87	26.14656468	14.26708786	24.19000764	8.1269549156E-02
18.88	26.15772026	14.27605199	24.19835780	8.1235934297E-02
18.89	26.16887526	14.28501658	24.20670678	8.1202350296E-02
18.90	26.18002967	14.29398164	24.21505459	8.1168797107E-02
18.91	26.19118350	14.30294717	24.22340122	8.1135274686E-02
18.92	26.20233676	14.31191317	24.23174668	8.1101782990E-02
18.93	26.21348943	14.32087963	24.24009097	8.1068321975E-02
18.94	26.22464152	14.32984655	24.24843408	8.1034891597E-02
18.95	26.23579303	14.33881395	24.25677602	8.1001491811E-02
18.96	26.24694397	14.34778180	24.26511679	8.0968122575E-02
18.97	26.25809433	14.35675013	24.27345639	8.0934783845E-02
18.98	26.26924411	14.36571891	24.28179482	8.0901475577E-02
18.99	26.28039331	14.37468816	24.29013209	8.0868197729E-02
19.00	26.29154194	14.38365788	24.29846819	8.0834950256E-02
19.01	26.30268999	14.39262805	24.30680312	8.0801733116E-02
19.02	26.31383747	14.40159869	24.31513688	8.0768546265E-02
19.03	26.32498438	14.41056979	24.32346949	8.0735389660E-02
19.04	26.33613071	14.41954135	24.33180092	8.0702263259E-02
19.05	26.34727646	14.42851338	24.34013120	8.0669167018E-02
19.06	26.35842165	14.43748586	24.34846032	8.0636100896E-02
19.07	26.36956626	14.44645880	24.35678827	8.0603064848E-02
19.08	26.38071031	14.45543221	24.36511506	8.0570058832E-02

19.09	26.39185378	14.46440607	24.37344070	8.0537082806E-02
19.10	26.40299668	14.47338040	24.38176518	8.0504136728E-02
19.11	26.41413901	14.48235518	24.39008850	8.0471220555E-02
19.12	26.42528077	14.49133042	24.39841066	8.0438334244E-02
19.13	26.43642197	14.50030612	24.40673167	8.0405477753E-02
19.14	26.44756260	14.50928227	24.41505152	8.0372651041E-02
19.15	26.45870265	14.51825889	24.42337022	8.0339854066E-02
19.16	26.46984215	14.52723596	24.43168777	8.0307086784E-02
19.17	26.48098107	14.53621348	24.44000417	8.0274349155E-02
19.18	26.49211943	14.54519146	24.44831941	8.0241641136E-02
19.19	26.50325723	14.55416990	24.45663351	8.0208962686E-02
19.20	26.51439446	14.56314879	24.46494645	8.0176313763E-02
19.21	26.52553113	14.57212814	24.47325825	8.0143694326E-02
19.22	26.53666723	14.58110794	24.48156890	8.0111104333E-02
19.23	26.54780278	14.59008819	24.48987840	8.0078543742E-02
19.24	26.55893775	14.59906890	24.49818676	8.0046012513E-02
19.25	26.57007217	14.60805006	24.50649397	8.0013510603E-02
19.26	26.58120603	14.61703168	24.51480004	7.9981037973E-02
19.27	26.59233932	14.62601374	24.52310497	7.9948594580E-02
19.28	26.60347206	14.63499626	24.53140875	7.9916180384E-02
19.29	26.61460424	14.64397923	24.53971139	7.9883795344E-02
19.30	26.62573585	14.65296265	24.54801289	7.9851439418E-02
19.31	26.63686691	14.66194652	24.55631325	7.9819112567E-02
19.32	26.64799742	14.67093084	24.56461248	7.9786814749E-02
19.33	26.65912736	14.67991561	24.57291056	7.9754545924E-02
19.34	26.67025675	14.68890083	24.58120751	7.9722306051E-02
19.35	26.68138558	14.69788650	24.58950332	7.9690095089E-02
19.36	26.69251386	14.70687262	24.59779800	7.9657912999E-02
19.37	26.70364158	14.71585919	24.60609154	7.9625759740E-02
19.38	26.71476874	14.72484620	24.61438394	7.9593635272E-02
19.39	26.72589536	14.73383366	24.62267522	7.9561539554E-02
19.40	26.73702142	14.74282157	24.63096536	7.9529472547E-02
19.41	26.74814692	14.75180993	24.63925437	7.9497434210E-02
19.42	26.75927188	14.76079873	24.64754226	7.9465424503E-02
19.43	26.77039628	14.76978797	24.65582901	7.9433443387E-02
19.44	26.78152013	14.77877767	24.66411463	7.9401490822E-02
19.45	26.79264343	14.78776780	24.67239913	7.9369566768E-02
19.46	26.80376618	14.79675839	24.68068250	7.9337671186E-02
19.47	26.81488838	14.80574941	24.68896474	7.9305804035E-02
19.48	26.82601003	14.81474088	24.69724586	7.9273965277E-02
19.49	26.83713114	14.82373280	24.70552585	7.9242154871E-02
19.50	26.84825169	14.83272515	24.71380472	7.9210372780E-02
19.51	26.85937170	14.84171795	24.72208247	7.9178618962E-02
19.52	26.87049116	14.85071119	24.73035910	7.9146893380E-02
19.53	26.88161008	14.85970488	24.73863460	7.9115195994E-02
19.54	26.89272844	14.86869900	24.74690898	7.9083526764E-02
19.55	26.90384627	14.87769357	24.75518225	7.9051885653E-02
19.56	26.91496355	14.88668857	24.76345440	7.9020272621E-02
19.57	26.92608028	14.89568402	24.77172543	7.8988687629E-02
19.58	26.93719647	14.90467991	24.77999534	7.8957130638E-02
19.59	26.94831212	14.91367624	24.78826413	7.8925601611E-02
19.60	26.95942723	14.92267300	24.79653182	7.8894100507E-02
19.61	26.97054179	14.93167021	24.80479838	7.8862627289E-02
19.62	26.98165581	14.94066785	24.81306384	7.8831181919E-02
19.63	26.99276929	14.94966593	24.82132818	7.8799764357E-02
19.64	27.00388223	14.95866445	24.82959141	7.8768374565E-02
19.65	27.01499463	14.96766341	24.83785352	7.8737012506E-02
19.66	27.02610649	14.97666280	24.84611453	7.8705678141E-02
19.67	27.03721781	14.98566263	24.85437443	7.8674371432E-02
19.68	27.04832860	14.99466289	24.86263322	7.8643092341E-02
19.69	27.05943884	15.00366359	24.87089090	7.8611840830E-02
19.70	27.07054855	15.01266473	24.87914748	7.8580616860E-02

19.71	27.08165772	15.02166630	24.88740295	7.8549420395E-02
19.72	27.09276636	15.03066831	24.89565731	7.8518251397E-02
19.73	27.10387446	15.03967075	24.90391057	7.8487109827E-02
19.74	27.11498202	15.04867362	24.91216273	7.8455995649E-02
19.75	27.12608905	15.05767693	24.92041378	7.8424908824E-02
19.76	27.13719554	15.06668067	24.92866373	7.8393849315E-02
19.77	27.14830151	15.07568484	24.93691258	7.8362817085E-02
19.78	27.15940693	15.08468945	24.94516033	7.8331812096E-02
19.79	27.17051183	15.09369449	24.95340698	7.8300834312E-02
19.80	27.18161619	15.10269996	24.96165253	7.8269883695E-02
19.81	27.19272002	15.11170586	24.96989698	7.8238960207E-02
19.82	27.20382332	15.12071219	24.97814034	7.8208063812E-02
19.83	27.21492609	15.12971895	24.98638260	7.8177194473E-02
19.84	27.22602833	15.13872614	24.99462376	7.8146352153E-02
19.85	27.23713004	15.14773376	25.00286383	7.8115536815E-02
19.86	27.24823122	15.15674181	25.01110281	7.8084748423E-02
19.87	27.25933188	15.16575029	25.01934069	7.8053986938E-02
19.88	27.27043200	15.17475920	25.02757748	7.8023252326E-02
19.89	27.28153160	15.18376854	25.03581318	7.7992544549E-02
19.90	27.29263067	15.19277830	25.04404779	7.7961863571E-02
19.91	27.30372921	15.20178849	25.05228131	7.7931209356E-02
19.92	27.31482723	15.21079911	25.06051374	7.7900581866E-02
19.93	27.32592472	15.21981016	25.06874508	7.7869981066E-02
19.94	27.33702169	15.22882163	25.07697533	7.7839406919E-02
19.95	27.34811813	15.23783353	25.08520450	7.7808859390E-02
19.96	27.35921405	15.24684585	25.09343258	7.7778338442E-02
19.97	27.37030944	15.25585860	25.10165957	7.7747844039E-02
19.98	27.38140431	15.26487177	25.10988548	7.7717376146E-02
19.99	27.39249866	15.27388537	25.11811031	7.7686934725E-02
20.00	27.40359249	15.28289939	25.12633406	7.7656519742E-02

Table of the INTEGRALS of the functions:
wzl(1/x), 1/wzl(1/x), wzl(x)/x and 1/[x*wzl(x)]
These integrals were first computed in closed form in April 1983
to answer questions about convergence and classes of functions.

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x	V(x)	S(x)	D(x)	-car(x)
.01	1.7555794993E-02	9.6634075233E-05	1.407414036	2.6262783271E-02
.02	3.0355307319E-02	3.3805171526E-04	1.180092766	4.0504588452E-02
.03	4.1468462113E-02	6.9823314349E-04	1.051815691	5.0965118932E-02
.04	5.1520722284E-02	1.1639149853E-03	.9632343015	5.9268165545E-02
.05	6.0803552351E-02	1.7260950898E-03	.8960613667	6.6139026756E-02
.06	6.9486117294E-02	2.3779912801E-03	.8422526621	7.1980244719E-02
.07	7.7678817572E-02	3.1141880438E-03	.7975613896	7.7042794616E-02
.08	8.5459266945E-02	3.9301956324E-03	.7594753156	8.1495207254E-02
.09	9.2884927725E-02	4.8221921933E-03	.7263869937	8.5456695404E-02
.10	.1000000000	5.7868598938E-03	.6972068934	8.9014919081E-02
.11	.1068394978	6.8212743364E-03	.6711631435	9.2236309863E-02
.12	.1134318145	7.9228264536E-03	.6476894125	9.5172445649E-02
.13	.1198004171	9.0891653263E-03	.6263581689	9.7864178182E-02
.14	.1259650103	1.0318155088E-02	.6068389481	.1003444099
.15	.1319423619	1.1607841651E-02	.5888711756	.1026400222
.16	.1377469044	1.2956426469E-02	.5722458579	.1047732504
.17	.1433911836	1.4362245676E-02	.5567928865	.1067626865
.18	.1488861999	1.5823752758E-02	.5423720105	.1086240257
.19	.1542416722	1.7339504657E-02	.5288662787	.1103706307
.20	.1594662459	1.8908149981E-02	.5161771825	.1120139658
.21	.1645676576	2.0528419151E-02	.5042209984	.1135639343
.22	.1695528687	2.2199115892E-02	.4929259906	.1150291451
.23	.1744281741	2.3919110225E-02	.4822302437	.1164171246

.24	.1791992911	2.5687332127E-02	.4720799623	.1177344876
.25	.1838714341	2.7502766177E-02	.4624281212	.1189870757
.26	.1884493766	2.9364446847E-02	.4532333850	.1201800706
.27	.1929375033	3.1271454337E-02	.4444592329	.1213180879
.28	.1973398548	3.3222911012E-02	.4360732457	.1224052550
.29	.2016601654	3.5217978003E-02	.4280465191	.1234452753
.30	.2059018962	3.7255852421E-02	.4203531781	.1244414838
.31	.2100682625	3.9335764741E-02	.4129699719	.1253968926
.32	.2141622593	4.1456976466E-02	.4058759338	.1263142304
.33	.2181866817	4.3618778034E-02	.3990520949	.1271959760
.34	.2221441443	4.5820486920E-02	.3924812401	.1280443874
.35	.2260370974	4.8061445925E-02	.3861477013	.1288615261
.36	.2298678416	5.0341021536E-02	.3800371787	.1296492791
.37	.2336385406	5.2658602614E-02	.3741365881	.1304093770
.38	.2373512330	5.5013599000E-02	.3684339282	.1311434109
.39	.2410078421	5.7405440337E-02	.3629181659	.1318528462
.40	.2446101853	5.9833574964E-02	.3575791359	.1325390351
.41	.2481599825	6.2297468898E-02	.3524074530	.1332032281
.42	.2516588632	6.4796604900E-02	.3473944353	.1338465829
.43	.2551083732	6.7330481601E-02	.3425320367	.1344701739
.44	.2585099810	6.9898612698E-02	.3378127868	.1350749989
.45	.2618650829	7.2500526208E-02	.3332297377	.1356619870
.46	.2651750082	7.5135763759E-02	.3287764178	.1362320036
.47	.2684410235	7.7803880000E-02	.3244467889	.1367858566
.48	.2716643374	8.0504441902E-02	.3202352094	.1373243009
.49	.2748461036	8.3237028264E-02	.3161364007	.1378480429
.50	.2779874248	8.6001229165E-02	.3121454170	.1383577444
.51	.2810893559	8.8796645467E-02	.3082576183	.1388540261
.52	.2841529066	9.1622888354E-02	.3044686462	.1393374712
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7.92	.7938915329	4.884947611	1.8797374832E-02	.1810845436
7.93	.7940793321	4.892887465	1.8762473958E-02	.1810952274
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7.96	.7946406424	4.916718273	1.8658363774E-02	.1811271671
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17.07	.8872077415	12.66161565	5.5552918163E-03	.1867366107
17.08	.8872632670	12.67048800	5.5497934495E-03	.1867403446
17.09	.8873187374	12.67936091	5.5443032870E-03	.1867440755
17.10	.8873741531	12.68823438	5.5388213124E-03	.1867478035
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17.13	.8875400716	12.71485809	5.5224243526E-03	.1867589696
17.14	.8875952685	12.72373377	5.5169749664E-03	.1867626857
17.15	.8876504111	12.73261000	5.5115336867E-03	.1867663990
17.16	.8877054992	12.74148678	5.5061004973E-03	.1867701092
17.17	.8877605331	12.75036411	5.5006753822E-03	.1867738166
17.18	.8878155128	12.75924199	5.4952583252E-03	.1867775210
17.19	.8878704383	12.76812042	5.4898493102E-03	.1867812225
17.20	.8879253098	12.77699940	5.4844483214E-03	.1867849211
17.21	.8879801273	12.78587892	5.4790553427E-03	.1867886168
17.22	.8880348909	12.79475900	5.4736703582E-03	.1867923095
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17.24	.8881442568	12.81252079	5.4629243085E-03	.1867996864
17.25	.8881988593	12.82140251	5.4575632116E-03	.1868033704
17.26	.8882534081	12.83028477	5.4522100456E-03	.1868070516
17.27	.8883079035	12.83916757	5.4468647948E-03	.1868107299
17.28	.8883623454	12.84805093	5.4415274436E-03	.1868144053
17.29	.8884167341	12.85693482	5.4361979762E-03	.1868180778
17.30	.8884710694	12.86581926	5.4308763772E-03	.1868217474
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17.32	.8885795807	12.88358977	5.4202567218E-03	.1868290781
17.33	.8886337568	12.89247583	5.4149586344E-03	.1868327391
17.34	.8886878799	12.90136244	5.4096683532E-03	.1868363973
17.35	.8887419502	12.91024959	5.4043858629E-03	.1868400526
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17.87	.8914826407	13.37311394	5.1400625433E-03	.1870262859
17.88	.8915340169	13.38202902	5.1351718794E-03	.1870297952
17.89	.8915853442	13.39094462	5.1302882281E-03	.1870333019
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17.91	.8916878524	13.40877735	5.1205419091E-03	.1870403072
17.92	.8917390335	13.41769449	5.1156792146E-03	.1870438059
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17.98	.8920451021	13.47120802	5.0866487176E-03	.1870647426
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18.90	.8965294856	14.29398164	4.6709147366E-03	.1873743863
18.91	.8965761736	14.30294717	4.6666809639E-03	.1873776395
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19.03	.8971331456	14.41056979	4.6163235650E-03	.1874164967
19.04	.8971792881	14.41954135	4.6121641213E-03	.1874197199
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19.06	.8972714483	14.43748586	4.6038621585E-03	.1874261593
19.07	.8973174662	14.44645880	4.5997196190E-03	.1874293756
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19.13	.8975927062	14.50030612	4.5749819259E-03	.1874486256
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19.17	.8977753776	14.53621348	4.5586013358E-03	.1874614137
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19.19	.8978664680	14.55416990	4.5504441614E-03	.1874677942
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19.23	.8980481605	14.59008819	4.5341956560E-03	.1874805284
19.24	.8980934822	14.59906890	4.5301471894E-03	.1874837063
19.25	.8981387635	14.60805006	4.5261041670E-03	.1874868821
19.26	.8981840043	14.61703168	4.5220665790E-03	.1874900555
19.27	.8982292048	14.62601374	4.5180344157E-03	.1874932268
19.28	.8982743650	14.63499626	4.5140076672E-03	.1874963958
19.29	.8983194850	14.64397923	4.5099863238E-03	.1874995627
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19.36	.8986342026	14.70687262	4.4819874510E-03	.1875216684
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19.38	.8987237628	14.72484620	4.4740358693E-03	.1875279646
19.39	.8987684833	14.73383366	4.4700680466E-03	.1875311093
19.40	.8988131641	14.74282157	4.4661055235E-03	.1875342519
19.41	.8988578054	14.75180993	4.4621482904E-03	.1875373923
19.42	.8989024071	14.76079873	4.4581963378E-03	.1875405305
19.43	.8989469693	14.76978797	4.4542496564E-03	.1875436665
19.44	.8989914921	14.77877767	4.4503082367E-03	.1875468004
19.45	.8990359755	14.78776780	4.4463720693E-03	.1875499320

19.46	.8990804196	14.79675839	4.4424411449E-03	.1875530615
19.47	.8991248244	14.80574941	4.4385154540E-03	.1875561888
19.48	.8991691899	14.81474088	4.4345949875E-03	.1875593140
19.49	.8992135163	14.82373280	4.4306797359E-03	.1875624370
19.50	.8992578035	14.83272515	4.4267696900E-03	.1875655578
19.51	.8993020517	14.84171795	4.4228648405E-03	.1875686765
19.52	.8993462608	14.85071119	4.4189651782E-03	.1875717930
19.53	.8993904310	14.85970488	4.4150706939E-03	.1875749073
19.54	.8994345623	14.86869900	4.4111813782E-03	.1875780195
19.55	.8994786547	14.87769357	4.4072972222E-03	.1875811296
19.56	.8995227082	14.88668857	4.4034182166E-03	.1875842374
19.57	.8995667230	14.89568402	4.3995443522E-03	.1875873432
19.58	.8996106991	14.90467991	4.3956756199E-03	.1875904468
19.59	.8996546366	14.91367624	4.3918120107E-03	.1875935483
19.60	.8996985354	14.92267300	4.3879535155E-03	.1875966476
19.61	.8997423957	14.93167021	4.3841001251E-03	.1875997448
19.62	.8997862174	14.94066785	4.3802518306E-03	.1876028399
19.63	.8998300007	14.94966593	4.3764086230E-03	.1876059328
19.64	.8998737456	14.95866445	4.3725704932E-03	.1876090236
19.65	.8999174521	14.96766341	4.3687374322E-03	.1876121123
19.66	.8999611204	14.97666280	4.3649094311E-03	.1876151989
19.67	.9000047503	14.98566263	4.3610864810E-03	.1876182833
19.68	.9000483421	14.99466289	4.3572685729E-03	.1876213656
19.69	.9000918957	15.00366359	4.3534556979E-03	.1876244458
19.70	.9001354112	15.01266473	4.3496478472E-03	.1876275239
19.71	.9001788887	15.02166630	4.3458450118E-03	.1876305999
19.72	.9002223282	15.03066831	4.3420471831E-03	.1876336738
19.73	.9002657297	15.03967075	4.3382543520E-03	.1876367456
19.74	.9003090933	15.04867362	4.3344665098E-03	.1876398153
19.75	.9003524190	15.05767693	4.3306836478E-03	.1876428828
19.76	.9003957070	15.06668067	4.3269057572E-03	.1876459483
19.77	.9004389571	15.07568484	4.3231328292E-03	.1876490117
19.78	.9004821696	15.08468945	4.3193648551E-03	.1876520730
19.79	.9005253445	15.09369449	4.3156018261E-03	.1876551322
19.80	.9005684817	15.10269996	4.3118437337E-03	.1876581894
19.81	.9006115814	15.11170586	4.3080905691E-03	.1876612444
19.82	.9006546435	15.12071219	4.3043423238E-03	.1876642973
19.83	.9006976682	15.12971895	4.3005989889E-03	.1876673482
19.84	.9007406555	15.13872614	4.2968605561E-03	.1876703970
19.85	.9007836054	15.14773376	4.2931270165E-03	.1876734438
19.86	.9008265181	15.15674181	4.2893983618E-03	.1876764884
19.87	.9008693934	15.16575029	4.2856745833E-03	.1876795310
19.88	.9009122316	15.17475920	4.2819556725E-03	.1876825715
19.89	.9009550326	15.18376854	4.2782416209E-03	.1876856100
19.90	.9009977964	15.19277830	4.2745324199E-03	.1876886464
19.91	.9010405232	15.20178849	4.2708280612E-03	.1876916807
19.92	.9010832130	15.21079911	4.2671285362E-03	.1876947130
19.93	.9011258658	15.21981016	4.2634338365E-03	.1876977432
19.94	.9011684817	15.22882163	4.2597439537E-03	.1877007714
19.95	.9012110607	15.23783353	4.2560588793E-03	.1877037975
19.96	.9012536029	15.24684585	4.2523786050E-03	.1877068216
19.97	.9012961083	15.25585860	4.2487031225E-03	.1877098436
19.98	.9013385770	15.26487177	4.2450324233E-03	.1877128636
19.99	.9013810090	15.27388537	4.2413664991E-03	.1877158816
20.00	.9014234043	15.28289939	4.2377053417E-03	.1877188975

V(x) = 1/wzl(1/x)
S(x) = ING[0,x,du/wzl(1/u)]
D(x) = d/dx[V(x)]
car(x) = x*wzl(x)*ei{-2*ln[wzl(1/x)]}

We hope you have enjoyed our "book"! Thank you for reading it.